ANALYSIS OF OBLIQUE COLLISION BETWEEN PARALLEL-PLANE MOVING CONSTRAINED BODY AND OBSTACLE

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Abstract. In everyday life, in engineering technology and processes, as well as in mechanical engineering, oblique collisions of bodies are widespread. These collisions can often be analysed numerically using computer programs. However, in many cases involving oblique collision analysis, optimization, and synthesis, it is desirable to derive analytical relationships in a general form. Such analysis can help predict desirable outcomes (e.g. rapid rebound) or prevent undesirable ones (e.g. jamming). This study focuses on deriving analytical relationships using classical mechanics theorems (such as changes in linear and angular momentum) and fundamental principles of physics (including sliding and rolling friction). First, the basic problem of a plane oblique collision between a wheel (or disk-shaped object) and a stationary plane is examined. The derived relationships align with well-established results in shock theory. The second part of the study extends to oblique collisions involving segment-shaped bodies. Analytical equations are derived for specific collision scenarios, which can be applied in further dynamic studies. The third part of the work examines the jumping and collision motion of a segment-type object along an inclined slope. Finally, the study includes computer modelling examples of oblique collisions, performed using the Working Model program, which validate the theoretical findings and demonstrate their practical applicability.

Keywords: collision, impulses, restitution, solid body.

Introduction

The study of close interactions between objects, particularly when a gap between them exists before collision, became widespread during the development of vibration transport technology [1]. As the efficiency of vibratory transport increased, leading to the periodic jumping of the transported object along the track, various hypotheses were proposed to investigate this process [2]. Expectedly, it was confirmed that all the laws of classical mechanics apply to the collision process. Thus, when describing an impact, several possible scenarios can be considered at the point of contact, such as [3-6]:

- full sliding in one direction;
- sliding stops during the first or second phase of a normal pulse;
- sliding in two opposite directions.

The conducted studies demonstrate that the collision process can be described comprehensively using the normal impulse restitution coefficient, the dry friction equilibrium region, and the boundaries of this region. Analysis of the results of the above studies shows that a planar collision between an object and a stationary obstacle can be classified into seven distinct cases [7]. These cases are not theoretical hypotheses but depend on factors such as the mass, position, and geometry of the colliding object, as well as the coefficients of normal restitution and sliding friction, along with the initial conditions at the beginning of the collision [8]. As a result of these studies, collision processes have been effectively described, analyzed and applied in various engineering fields, playing a crucial role in numerous engineering disciplines, influencing calculations, technological advances, and experimental techniques [9-14]. Collision effects characterized by the transfer of energy and momentum are of great importance for applications in areas such as aerospace, automotive safety and material science.

Further research on collision processes is particularly focused on simultaneous impacts at multiple contact points [15-20]. However, not all possible collision scenarios have been fully explored. For instance, the effects of rolling friction during impact and the influence of additional constraints on the collision process require further investigation. This paper focuses on addressing some of these unresolved issues with the aim of deriving analytical relations using theorems of classical mechanics and fundamental principles of physics [21].

Collision models and methods of analysis

At first glance, the process of a planar collision between an object and a stationary obstacle may seem simple, but it is quite complex. The collision dynamics depend on approximately ten parameters, including the object's mass, its axial moment of inertia of the centre of mass, the coordinates of the collision point, and the object's orientation angle. Additionally, in three-dimensional motion space, the process is influenced by two components of the centre of mass velocity, the angular velocity of rotation, and the coefficients of sliding and rolling friction.

The interaction of these ten parameters gives rise to various collision scenarios. However, only three final parameters ultimately characterize the outcome: the two components of the centre of mass velocity and the angular velocity after impact. These three parameters are critical for effective analysis, optimization and synthesis of impact processes. The following sections present our new analytical calculations and numerical simulation examples that illustrate the collision dynamics in more detail.

Plane oblique collision of a wheel with a plane

The simplest case of collision, in which a wheel (with no rotational angular velocity, $\omega = 0$) impacts a stationary plane with an initial velocity V_0 along the normal O*y* direction, is shown in Fig. 1.

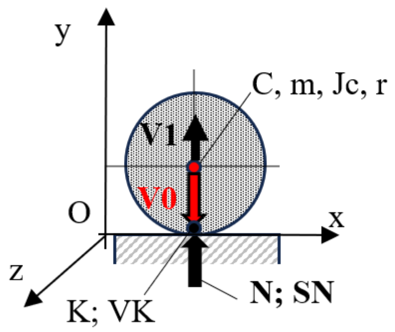

Fig. 1. **Straight central impact of a disc against a stationary plane:** *C* – centre of mass; *m* – disc mass; J_c – moment of inertia of the mass against the C_z axis; *r* – radius; *N* – normal force

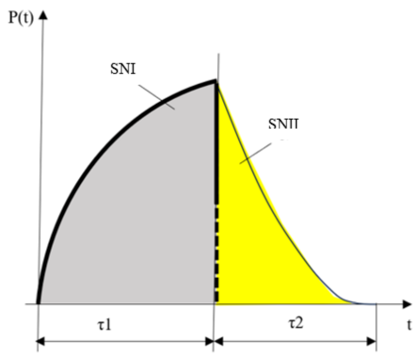

Fig. 2. **Impact force P(t) as a function of time t:** τ_1, τ_2 – durations of the first and second phases of impact, respectively; SNI – full impulse of the first phase of the impact, as an area; SNII – full impulse of the second phase of the impact, as an area

This collision process can be described using the integral form of the disc's momentum change over two impact time periods τ_1 and τ_2, as follows [7; 8; 21]:

$$0 - (-mV_0) = SNI;$$
$$mV_1 - 0 = SNII; \qquad (1)$$
$$SNII = R \cdot SNI;$$

where $SNI = \int_0^{\tau 1} P(t)\,dt$ – full impulse of the impact force *P*(*t*) in the first impact period τ_1;
$SNII = \int_{\tau 1}^{\tau 1 + \tau 2} P(t)\,dt$ – full impulse of the force *P*(*t*) in the second impact period τ_2;
m – mass of the disc;
V_1 – disc velocity after collision;
R – coefficient of restitution of normal impulse *SNI*.

Plot of the impact force *P*(*t*) as a function of time *t* is shown in Fig. 2. From the three expressions (1), all the unknown quantities V_1, SNI and $SNII$ can be determined and subsequently applied to the analysis of the oblique collision discussed below.

Oblique central collision of a wheel with a plane

The simplest oblique impact process occurs when the disk does not rotate before the collision ($\omega_0 = 0$), but has an additional tangential component of velocity u_0 along the O*x* axis (Fig. 3). In this case, during the impact, a tangential force of interaction *F* and a rolling moment $M_z(\delta)$ arise. As a result of this interaction, changes in the linear momentum along the O*x* axis and in the angular momentum about the center of mass can be expressed as follows [21]:

$$mu_1 - mu_0 = SF; \quad (2)$$

$$J_{Cz}\omega_1 - 0 = M_z^{(\delta)} - r \cdot SF; \quad (3)$$

$$M_z^{(\delta)} = d(SI + SII); \quad (4)$$

$$SI + SII = mV_0(1 + R), \quad (5)$$

where u_1 – component of velocity of center mass along Ox axis after collision;
SF – tangential full impulse;
J_{Cz} – moment of inertia;
$M_z^{(\delta)}$ – rolling resistance moment;
r – disk radius;
d – distance from the contact point K at which the rolling friction impulse moment $M_z^{(\delta)}$ is replaced by the normal impulse $SN = SI + SII$ (Fig. 3).

Plots of impact normal force $P(t)$ and tangential force $F(t)$ are shown in Fig. 4.

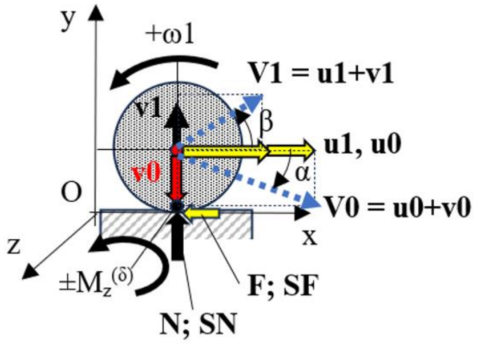

Fig. 3. **Oblique central impact model:** N – normal component of impact force; SN – impact full normal impulse; V_0 – initial velocity of center mass; u_0, v_0 – initial components of velocity of center mass along Ox and Oy axis; V_1 – velocity after collision; u_1, v_1 – components of velocity after rebound; F – tangential friction force; SF – tangential full impulse

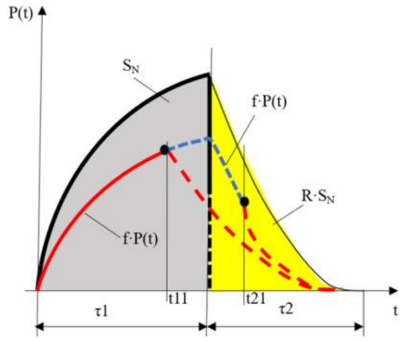

Fig. 4. **Impact force $P(t)$ and tangential force $F(t)$ as functions of time t:** τ_{11}, τ_{21} – moments of time of transformation of the tangential force $F(t)$ in the equilibrium region $|F(t)| \leq f \cdot P(t)$

The two equations (2)-(3) contain four unknowns: u_1, ω_1, SF and $M_z^{(\delta)}$. To determine the disc's post-collision parameters ω_1 and u_1, additional equations must be used to analyse possible collision scenarios [7; 8]. These cases and corresponding equations are presented in the following Table.

Table 1

Possible collision scenarios and corresponding equations

Collision scenario	Full or partial sliding (6)	Rolling rule (7)	Rule of existence
1.Full sliding and full rolling in u_0 direction	$SF = f(1 + R)mv_0$	$M_z^{(\delta)} = -\delta \cdot SF$	$u1 + \omega 1 \cdot r \geq 0$
2.Partial sliding and full rolling in u_0 direction (Fig. 4)	$u_1 + \omega_1 r = 0$	$M_z^{(\delta)} = -\delta \cdot SF$	$\lvert SF \rvert \leq f(1 + R)mv_0$
3. Full sliding and partial rolling in u_0 direction	$SF = f(1 + R)mv_0$	$\omega_1 = 0$	$\lvert d \rvert \leq \delta$
4. Getting stuck (Fig. 4)	$u_1 = 0$	$\omega_1 = 0$	$\lvert SF \rvert \leq f(1 + R)mv_0$; $\lvert d \rvert \leq \delta$

The analytical relations (2)-(7) enable the analysis and optimization of various collisions in a general manner without relying on numerical modelling, thereby reducing computation time. The simplified case of an oblique impact serves as an example of more complex collision scenarios, where sliding and rolling friction must also be considered. One such typical case involving a segment is discussed below.

Segmental body collision

In the collision process of a segment-type body, the positioning eccentricity h and the angle φ of the centre of mass C must be considered (Fig. 5). Additionally, the rule governing the change in direction of the normal component of the velocity VKy at the contact point K at the end of the first phase of the collision, when $t = \tau_1$, must be considered (Fig. 4 and Fig. 6):

$$VK_y = 0, \text{ or } \dot{Y} - \omega h \cdot \sin\varphi = 0, \quad (8)$$

where $\dot{Y}$ – projection of the velocity (v in Fig. 6) of the center C of mass at $t = \tau_1$;
ω – angular velocity of the segment at the time instant of $t = \tau_1$.

Accordingly, in the case of an eccentric collision, the direction of the sliding friction force F at the contact point K (whether in the Ox direction or its opposite) is determined by the following analytical expression for the projection VK_x of the velocity VK (Fig. 4 and Fig. 6):

$$\text{sign}(VK_x) = \text{sign}\left[\dot{X} + \omega(r - h \cdot \cos\varphi)\right], \quad (9)$$

where $\dot{X}$– projection of the velocity (u in Fig. 6) of the center C of mass at $t = \tau_1$.

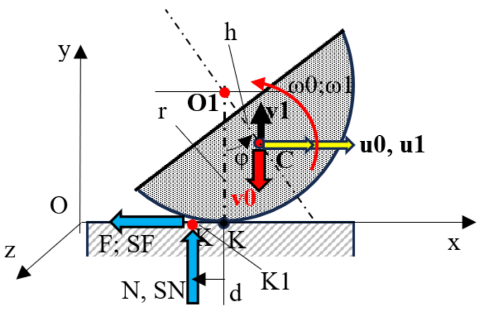

Fig. 5. **Additional parameters of segment body collision model:** h – eccentricity of the center of mass; φ – angular position of the center of mass; $K1$ – rolling friction moment reduction center; d – distance from the reduction center $K1$ to the contact point K

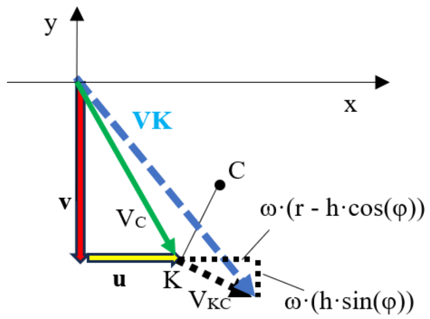

Fig. 6. **Velocity *VK* components at the contact point *K*:** V_C – velocity of the center of mass C; V_{kc} – velocity of relative rotation of the contact point K around the center of mass C

Since the segment rotational angular velocity ω can change direction during the collision process, the number of possible motion scenarios increases significantly. When analyzing these cases, the relationships governing changes in both linear momentum and angular momentum should be applied at all points where the relay-type force change occurs, given by $F = \pm f \cdot N$ and the corresponding distance $d = \pm \delta$. Due to the extensive calculations required by the integral impact phase impulse method, this study instead utilizes differential equations to model the motion under given initial conditions. The segment's motion is described by the following three differential equations:

$$m\ddot{X} = F;$$
$$m\ddot{Y} = N; \quad (10)$$
$$J_{Cz}\ddot{\varphi} = -N[\delta \cdot \text{sign}\omega + h \cdot \sin\varphi] - F[r - h \cdot \cos\varphi].$$

The forces in equations (10) are defined by the following expressions:

$$F = -Nf \cdot \text{sign}(VK_x); \quad (11)$$

$$N = N0 \cdot \text{sign}[0.5 - 0.5 \cdot \text{sign}(Y_C + h \cdot \cos\varphi - r)]; \quad (12)$$

$$N0 = -c[1 - \Delta \cdot \mathrm{sign}(VK_y)] \cdot (Y_C + h \cdot \cos\varphi - r), \tag{13}$$

where $N0$ – normal component of the collision force, expressed as a linear function of the contact point deformation, with two stiffness values $c(1 + \Delta)$ and $c(1-\Delta)$;
Δ – stiffness parameter.

Results of computer simulation on the base of equations (10)-(13) are presented in the next section.

Results and discussion

A numerical simulation of the system dynamics, as described by equations (10)-(13), was conducted using the following parameter values: m = 1 kg; r = 0.1 m; δ = 0.001 m; h = 0.05 m; $J_{Cz} = 7.5 \cdot 10^{-3}$ kg·m^2; f = 0.1; $c = 9.81 \cdot 10^3$ kg·s^{-2}; Δ = 0.5; $x(0)$ = 0; $Vx(0)$ = 0; $y(0)$ = 0.07 m; $Vy(0)$ = -1 m·s^{-1}; $\varphi(0) = \pi \cdot 4^{-1}$; $\omega(0)$ = 0. The key graphs from the numerical modelling, generated using the MathCad software, are presented in Fig. 7-10. The Euler integration step Δt was chosen as follows: $\Delta t = 2.5 \cdot 10^{-7}$ s.

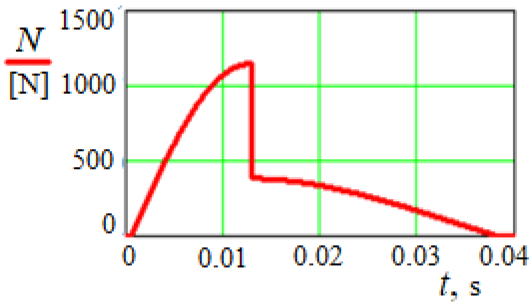

Fig. 7. **Change in time *t* of the normal component *N* of the collision force**

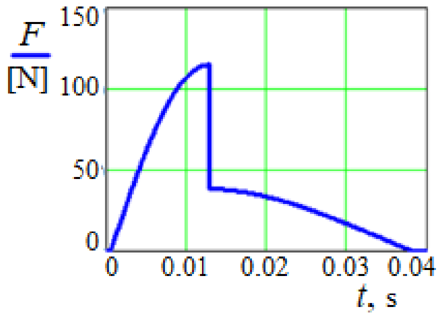

Fig. 8. **Change in time *t* of the tangential component *F* of the collision force**

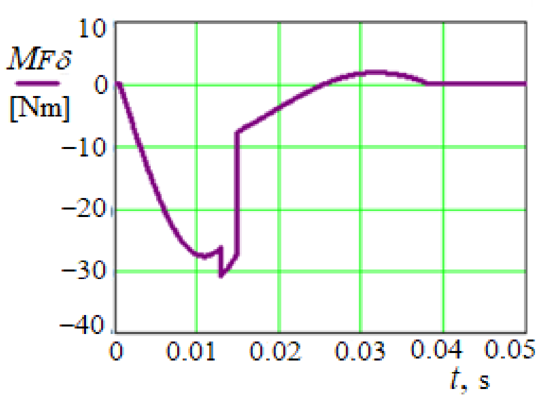

Fig. 9. **Change in time *t* of the total moment of the collision forces *N*, *F* and rolling friction**

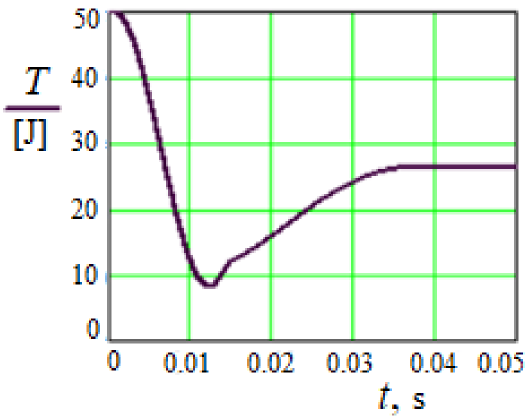

Fig. 10. **Change in time *t* of the kinetic energy:** $T(0)$ = 50 J; $T(0.04)$ = 26.43 J

The following conclusions can be drawn from the simulation results (Fig. 7-10):

- numerical simulation enables determining the impact time when the approximate local interaction stiffness c is specified;
- the selected normal interaction force model effectively accounts for energy losses, preventing objects from sticking together;
- as shown in Fig. 8, complete sliding occurred at the contact point during the collision, and in the first phase of the collision, the rolling friction torque changed direction (Fig. 9);
- the proposed collision model can simulate successive repeated collisions if the interaction forces between impacts are incorporated into differential equations (10).

Successive repeated collisions are further analyzed using the example of a sled descending an inclined slope (Fig. 11), considering segmental body motion. The simulation was conducted using the computer program Working Model 2D, with the following main system parameters: mass m = 100 kg; coefficient of sliding friction against ice f = 0.03; coefficient of elasticity 0.5.

Simulation results are shown graphically in Figures 12-14. These simulations of oblique collisions validate the theoretical findings and illustrate their practical applicability.

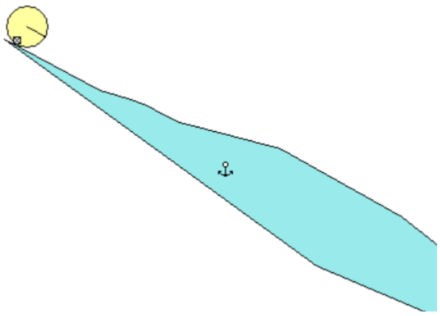

Fig. 11. **Object on an inclined plane**

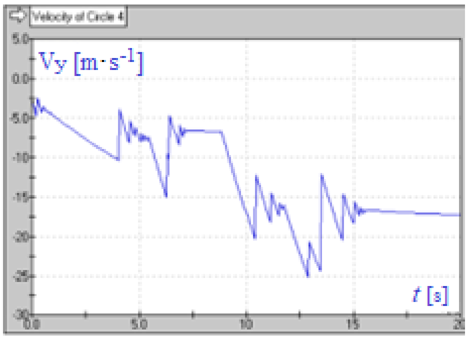

Fig. 12. **Change in time *t* of the vertical component of velocity**

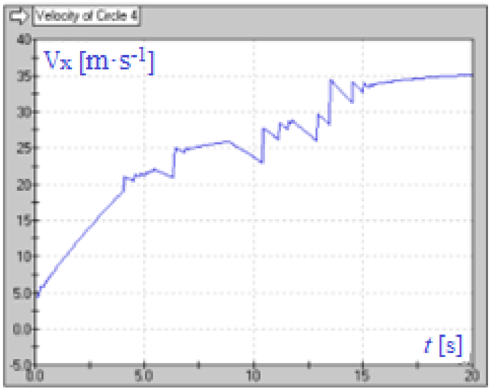

Fig. 13. **Change in time *t* of the horizontal component of velocity**

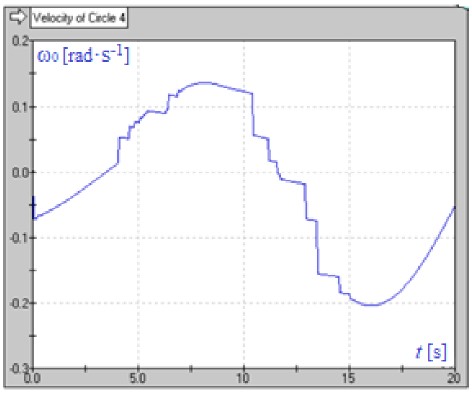

Fig. 14. **Change in time *t* of the angular velocity**

Today, the dynamics of complex objects are typically analyzed using numerical methods, which rely on analytical descriptions to clarify how motion evolves from initial conditions. This study extends collision analysis by incorporating rolling friction, offering deeper insight into collision processes involving changes in rotational momentum.

Conclusions

1. A new model for the normal component of the collision force has been developed, preventing objects from sticking during multiple collisions.
2. The existing cases of sliding and sliding-stopping collisions have been extended to include the effects of rolling friction.
3. It has been demonstrated that using discrete element models is advisable for analyzing repeated collisions.
4. Experimental studies involving eccentric oblique impacts – accounting for both sliding and rolling friction – are planned as part of the authors' future research on skeleton sled motion. To date, such experimental studies have not been thoroughly analyzed in the literature.

Author contributions

Conceptualization, J.V.; methodology, V.B. and E.K.; software, J.V.; validation, M.I. and V.B.; formal analysis, E.K. and M.E.; investigation, M.I., V.B., J.V. and E.K.; data curation, V.B.; writing – original draft preparation, V.B.; writing – review and editing, J.V. and V.B.; visualization, J.V.; project administration, V.B. All authors have read and agreed to the published version of the manuscript.

References

[1] Блехман И.И., Джанелидзе Г.Ю. Вибрационное перемещение (Vibratory movement). Москва: Наука, 1964. 412 p. (In Russian).

[2] Лавендел Э.Э. Система гипотез в технических расчетах по вибрационному перемещению (System of hypotheses in technical calculations for vibration displacement). In: Вопросы динамики и прочности (The Problems of Dynamics and Strength). Riga: Zinatne, 1971, issue 21, pp. 5-10. (In Russian).
[3] Плявниекс В.Ю. Расчет косого удара о препятствие (Calculation of oblique impact with an obstacle). In: Вопросы динамики и прочности (The Problems of Dynamics and Strength). Riga: Zinatne, 1969, issue 18, pp. 87-110. (In Russian).
[4] Плявниекс В.Ю. Косое соударение двух тел (Oblique collision of two bodies). In: Вопросы динамики и прочности (The Problems of Dynamics and Strength). Riga: Zinatne, 1969, issue 19, pp. 69-82. (In Russian).
[5] Плявниекс В.Ю. Пространственное соударение с переменным направлением скольжения (Spatial collision with variable sliding direction). In: Вопросы динамики и прочности (The Problems of Dynamics and Strength). Riga: Zinatne, 1971, issue 21, pp. 37-46. (In Russian).
[6] Sun H., Baruh H. Analysis of three-dimensional rigid-body impact with friction. Dynamics, 2(1), 2022, pp. 1-26. DOI: 10.3390/dynamics2010001
[7] Kepe O., Viba J. Teorētiskā mehānika (Theoretical Mechanics). Riga: Zvaigzne, 1982. 577 p. (In Latvian).
[8] Виба Я.А. Оптимизация и синтез виброударных машин (Optimization and Synthesis of Vibro-Impact Machines). Riga: Zinatne, 1988. 253 p. (In Russian).
[9] Taghavifar H., Motlagh A.M., Mardani A., Hassanpour A., Hosseinloo A.H., Wei C. The induced shock and impact force as affected by the obstacle geometric factors during tire-obstacle collision dynamics. Measurement, vol. 84, April 2016, pp. 47-55. DOI: 10.1016/j.measurement.2016.02.003
[10] Taghavifar H., Mardani A., Hosseinloo A.H. Experimental analysis of the dissipated energy through tire-obstacle collision dynamics. Energy, vol. 91 issue 11, 2015, pp. 573-578. DOI: 10.1016/j.energy.2015.08.050
[11] Punch O., Danczyk M., Hawken M., Holland D.J. A comparison of pendulum experiments and discrete-element simulations of oblique collisions of wet spheres. AIChE Journal, vol. 69 issue 3, 2023, pp. 1-13. DOI: 10.1002/aic.17989
[12] Hlosta J., Žurovec D., Rozbroj J., Ramírez-Gómez A., Nečas J., Zegzulka J. Experimental determination of particle–particle restitution coefficient via double pendulum method. Chemical Engineering Research and Design, vol. 135, 2018, pp. 222-233. DOI: 10.1016/j.cherd.2018.05.016
[13] Liang Z., Huang Y., Li D., Wada M.E. Parameter determination of a viscoelastic–plastic contact model for potatoes during transient collisions. Biosystems engineering, vol. 234, 2023, pp.156-171. DOI: 10.1016/j.biosystemseng.2023.09.003
[14] Dintwa E., Van Zeebroeck M., Ramon H., Tijskens E. Finite element analysis of the dynamic collision of apple fruit. Postharvest Biology and Technology, vol. 49, issue 2, 2008, pp. 260-276. DOI: 10.1016/j.postharvbio.2008.01.012
[15] Yu Wang, Matthew T. Mason. Two-Dimensional Rigid-Body Collisions with Friction. Journal of Applied Mechanics, vol. 59(3), 1992, pp. 635-642. DOI: 10.1115/1.2893771
[16] Dragna I-B., Pandrea N., Stănescu N-D. Simultaneous Collision of the Rigid Body at Two Points. Symmetry vol. 13(10), 2021, 1924. DOI: 10.3390/sym13101924
[17] Dal Pont S., Dimnet E. A theory for multiple collisions of rigid solids and numerical simulation of granular flow. International Journal of Solids and Structures, vol. 43, issue 20, 2006, pp. 6100–6114. DOI: 10.1016/j.ijsolstr.2005.10.001
[18] Viba J., Polukoshko S. Simultaneous collisions of rigid bodies in more than one point. Proceedings of the 4th International DAAAM Conference "Industrial Engineering – Innovation as Competitive Edge for SME", April 29-30, 2004, Tallinn, Estonia, pp. 298-301.
[19] Kovals E., Viba J., Stals L., Sokolova S., Krupenin V. Mechanism motion studies with collisions at several points. Proceedings of the 15th International Scientific Conference "Engineering for Rural Development", May 25-27, 2016, Jelgava, Latvia, pp. 1104-1109.
[20] Warzecha M. An investigation of compliant contact force models applied for analysis of simultaneous, multi-zone impacts of particles. Computational Particle Mechanics, vol. 11, 2024, pp. 1-27. DOI: 10.1007/s40571-023-00606-w
[21] Meriam J.L., Kraige L.G., Bolton J.N. Engineering Mechanics: Dynamics. 8th edition. New York: John Wiley & Sons, 2016. 736 p.