INVESTIGATION OF PNEUMATIC TIRE OPERATION UNDER LOAD

Ramis Zaripov[1], Valeriy Yessaulkov[1], Pavels Gavrilovs[2]
[1]Toraighyrov University, Kazakhstan; [2]Riga Technical University, Latvia
ramis.zaripov@mail.ru, pavels.gavrilovs@rtu.lv

Abstract. Car wheels are operated in difficult conditions. The system of forces and torques acting on the wheel is determined by many factors: transfer of energy from the engine through the transmission to the wheel, environmental conditions, and the driver's control actions. During the work, previously developed and implemented tire calculation methods and experimental methods for operational tire testing were used. The purpose of the work is to determine the main design parameters for the theoretical study of a passenger car pneumatic tire under operating conditions and a mathematical model. The main calculated dependences of the stiffness, deflection and contact length parameters of the pneumatic tire of the vehicle are considered. The theory of thin flexible shells with large deflections was chosen to solve the research problem. The model of the crown part of the tire, with the upper part of the frame, the breaker and the tread is made in the form of a rigid in the transverse direction and flexible in the circumferential direction of the tape. The initial data for the calculation are the design parameters of the tire, air pressure and normal load. Laboratory data and many published papers show that when the wheel is loaded with a normal force, the stresses in the middle of the peripheral surface of the tire remain virtually unchanged, i.e. they are equal to the displacements and stresses caused by internal pressure. As a result of the work, the main results were obtained by calculation and experimentally. It is determined that under the influence of a normal load and the resulting displacement of the elements of the tire crown, the specific force decreases in the circumferential direction, which corresponds to the experimental data.

Keywords: tires, contact length, deflection, normal load, tire pressure.

Introduction

A car tire is an elastic shell located on the rim of the wheel. The tire is designed to absorb minor vibrations caused by imperfections in the road surface, realize and perceive the forces that arise in the contact patch with the road and ensure a high coefficient of adhesion.

The practical value of such studies lies in the fact that the obtained analytical dependences linking stresses and displacements in the contact zone with the actual design of the tire, the external forces transmitted through it, and the properties of materials provide an understanding of the mechanical processes in contact. They can be used in the development of methods for predicting wear resistance and rolling losses, introduced into the practice of tire design in enterprises.

Figure 1 shows the main structural elements of the tire (1 – frame; 2 – side wire; 3 – sideboard; 4 – sidewall; 5 – protector; 6 – breaker; 7 – filling cord), selected as the object of research, which developed the principles of modeling the processes of deformation and interaction with the supporting surface.

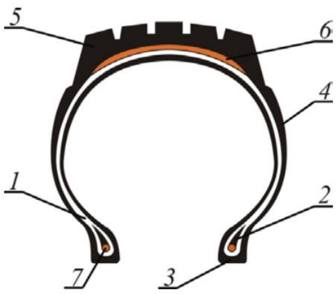

Fig.1. **Car tire construction**

Information about the stiffness and deformation characteristics of tires is necessary when creating refined calculated dynamic car models used to develop effective spring suspension systems. As a rule, research based on determining deformations and stresses in tire elements arising under the action of various types of loads results in complicated mathematical equations that are difficult to use for analysis and calculations.

In paper [1], the static load characteristics and pure longitudinal slip characteristics of the non-pneumatic tire are studied by combining experiments and simulations. The study [2] is devoted to composite pneumatic tires used in transportation equipment. The finite element method used for

DOI: 10.22616/ERDev.2025.24.TF004

numerical simulation, combined with preliminary experimental measurements, is based on optimizing the vibration characteristics of the material. In article [3], experiments and numerical simulations were carried out to study the properties of the static stiffness of a high-load tire. In the study [4], the sliding angle was calculated, which is an important parameter in the operation of safety and vehicle control systems, investigated mathematically, and a change in the transverse force and self -adjusting torque was obtained depending on the sliding angle for the load values on the axis.

The calculation in this paper did not take into account the influence of rubber on the stiffness characteristics of the tire, but this is the main focus of article [5] on the use of modeling to calculate the temperature and heat released during the distribution of flow in a tire moving at a constant speed using the finite element method.

Materials and methods

The coefficient of normal stiffness C_z, normal deflection h_z and contact length L are some of the main characteristics of a pneumatic tire. It is widely known that theoretical studies of the performance of a wheel equipped with a pneumatic tire are very difficult due to the anisotropy of the outer layer of the tire.

Let us consider the calculated dependencies of the parameters C_z, h_z and L on the normal load G_k, the internal pressure in the tire and the design parameters of the wheel with a pneumatic tire, obtained in an approximate form, and compare them with the experimental data.

The present work was carried out under laboratory conditions and was based on a mathematical model. The crown part of the tire, including the upper part of the frame, breaker and tread, is modeled as a band, rigid in the transverse direction and flexible in the circumferential direction. The theory of thin flexible shells of large deflections was used in solving the problem [6].

When considering the deformation of a tire on a flat support surface due to an applied normal load, we assume that the main load is borne by the cord thread when the sidewalls are deformed. Some non-flat curvature of the threads and changes in stress due to hysteresis can be neglected. The tread pattern saturation coefficient k is considered a continuous function, constant for any contact point. Let us consider the loads acting on the element of the crown part of the tire (band) during contact (Fig. 2). Let us designate the displacement of the middle surface of the element along the x and z axis as u and w, respectively. The following loads act on the element: tensile force N, bending moment M, shear force Q, force from the sidewall threads T, projections of force T on the x and z axes, respectively, shear forces Pc and Qc acting on the side faces of the element due to deformation of the rubber sidewalls, bending moment caused by deformation of the sidewall rubber Mc, normal and longitudinal tangential stresses acting from the tread rubber on the breaker and frame q and τ.

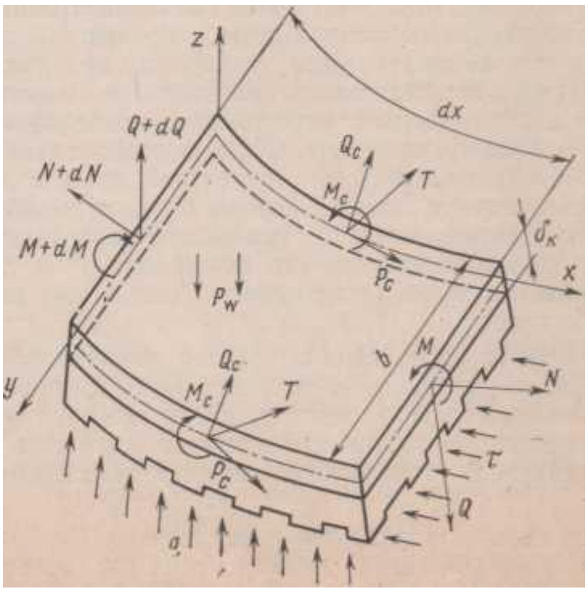

Fig. 2. **Loads acting on the band element of the model**

The basic laws used in the calculation were taken from the theory of thin shells [7].

Let us project all the forces sequentially on the *x* and *z* axes as well as the moments relative to the *y* axis and transform the resulting equations [8]:

$$\frac{dN}{dx} - T_x \frac{2i}{b} - \tau - P_c \frac{2}{b} = 0, \tag{1}$$

$$\frac{dQ}{dx} + N\left(\frac{1}{r_0} + \frac{d^2w}{d^2x}\right) - p_w + kq + T_z \frac{2l}{b} - Q_c \frac{2}{b} = 0, \tag{2}$$

$$Q = \frac{dM}{dx}, \tag{3}$$

where r_0 – free radius of the tire;
b – width of the tire tread;
i – density of cord threads along the crown in the circumferential direction.

The bending moment acting in the cross section is calculated using the equation:

$$M = -D\frac{d^2w}{dx^2}, \tag{4}$$

where D – cylindrical stiffness of the model band.

The calculation of the parameters N, D, Q_c and P_c is presented in the work [9]. The analysis of the kinematics of the cord threads during tire deformation under the action of a normal load makes it possible to obtain the following dependencies

$$T_x = -\varepsilon p_w t\left(r_0' \cos a_0 - a_1 w)\sin\frac{dw}{wx};\right), \tag{5}$$

$$T_x = \varepsilon p_w t(r_0' \cos a_0 - a_1 w)\sin\frac{dw}{wx}, \tag{6}$$

where ε – coefficient that takes into account the influence of the rubber of the sidewalls;
t – pitch of the frame threads;
r_0' and a_0 – average radius of thread curvature and half the girth angle of the undeformed sidewall thread;

$$a_1 = \frac{3\cos\beta_0}{a_0}\left(1 + \frac{a_1^2}{2}\right). \tag{7}$$

Assuming that the radial deformations of the blocks in the tire contact zone are insignificant relative to the deflection of the frame, we find that the bending moments in the circumferential direction in the crown of the tire along the entire length of the contact will be the same due to the constant curvature of the shell, that is

$$\frac{dM}{dx} = 0$$

and therefore

$$\frac{dQ}{dx} = 0$$

Since the contact area is flat, the curvature of the deformed crown part of the tire in the circumferential direction [10] equals zero. Based on this condition and taking into account that the movements must be symmetrical relative to the center of contact, we obtain:

$$w = -\frac{x^2}{2r_0} + h_z + w_0, \tag{8}$$

where w_0 – radial displacement due to the presence of internal pressure in the tire.

Substituting the known values into the equation (2) and finding the normal stress q from it, we integrate it over the entire contact area and obtain the expression for the normal load.

$$G_k = 2kb\int_0^{\frac{L}{2}} q dx = -\varepsilon_1 p_w bL\left[1 - a_2\left(h_z + w_0 - \frac{L^2}{24r^0}\right)\right] + \\ + 2k_c pL\left[h_z + w_0 - \frac{L^2}{24r^0}\right] + bp_w bL, \quad (9)$$

where $\varepsilon_1 = \varepsilon\frac{2}{b}r_0' \cos a_0$

$a_2 = \frac{a_1}{r_0' \cos a_0}$.

The contact length of a tire is its arithmetic mean value.

$$L = \frac{S}{b},$$

where S – imprint area of the tire along the contour.

The equilibrium equations (1) and (2) for the non-contact zone are outlined below:

$$\frac{dN}{dx} - T_x\frac{2i}{b} - P_c\frac{2}{b} = 0; \quad (10)$$

$$\frac{dQ}{dx} + N\left(\frac{1}{r^0} + \frac{d^2w}{dx^2}\right) - p_w + T_z\frac{2i}{b} - Q_c\frac{2}{b} = 0; \quad (11)$$

$$Q = \frac{dM}{dx}. \quad (12)$$

The laboratory data and many published papers indicate that when the wheel is loaded with a normal displacement force, the stresses in the middle of the peripheral crown of the tire (when $x = \pi r_0$) remain almost unchanged, i.e. equal to the displacements and stresses caused by the internal pressure (for radial tires, this assumption is somewhat rougher). Taking into account the latter assumption, we obtain by substituting the known values into expression (10) and solving it with respect to N

$$N \approx -\varepsilon_2 p_w + N_0 \quad (13)$$

where ε_2 – coefficient that takes into account the influence of the rubber of the sidewalls and some design parameters;

N_0 – tensile force of the crown part of the tire caused by the presence of internal pressure.

The last equation leads to that under the influence of a normal load and the resulting displacement of the elements of the crown part of the tire w, the specific force N decreases in the circumferential direction, which corresponds to the experimental data. Substituting known quantities, including the last expression for N into equation (11) and making the transformation, we obtain an inhomogeneous linear differential equation of the 4th order with constant coefficients. The general solution of this equation has the following form

$$w = C_1' l^{\lambda_1 x} + C_2' l^{-\lambda_1 x} + C_3' l^{\lambda_2 x} + C_4' l^{-\lambda_2 x} + w_0, \quad (14)$$

$$\lambda_{1,2} = \sqrt{\frac{N_0 \pm \sqrt{N_0^2 - 4a_3 D}}{2D}}. \quad (15)$$

The radial movements w of the crown of the tire must be symmetrical with respect to the vertical axis of the wheel:

$$w = \mathrm{C}_1 ch\lambda_1(x - \pi r_0) + \mathrm{C}_2 ch\lambda_2(x - \pi r_0) + w_0. (16) \quad (16)$$

Euqations (8) and (16) form a system of equations describing the displacements w for the contact and peripheral zones of the crown of the tire. It follows from the condition of their smooth conjugation at the contact boundary at $x = \frac{L}{2}$ the displacement w, the first and second derivatives must be equal for both zones. By equating them, we obtain a system of three equations, from which we find the values C_1, C_2 and the deflection value h_z of the tire [11-13].

The combined solution of equations (8), (9) and (16) gives the desired deflection values h_z of the tire and the contact length L for the specified design parameters, the values of the normal load G_k and the air pressure in the tire p_w (the equations for h_z and L are omitted due to cumbersomeness).

However, it is very difficult to determine the contact length L and deflection h_z based on these dependencies without resorting to computers.

Analysis of equation (16) shows that the effect on the value of w of the second term $C_2 ch\lambda_2(x - \pi r_0)$ is qualitatively similar to the first, but quantitatively insignificant, up to 7% of the total amount.

Let us omit this term, but correct the error by calculating the new constant C_1 in equation (16). Then the deflection value

$$h_z = \frac{L}{2r_0\lambda_1} + \frac{L^2}{8r_0}. \tag{17}$$

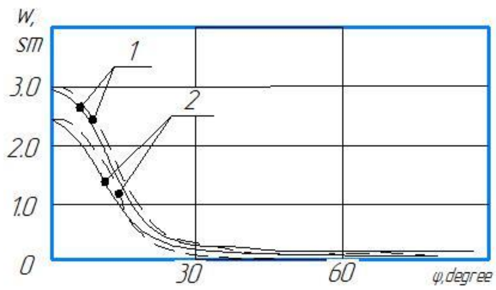

Fig. 3. **Radial movements of the crown part of a 260-20 tire in the driven mode at an air pressure of 5.5 kgf·cm^{-2} (curves 1) and 3.5 kgf·cm^{-2} (curves 2) at a normal load of 1860 kgf**

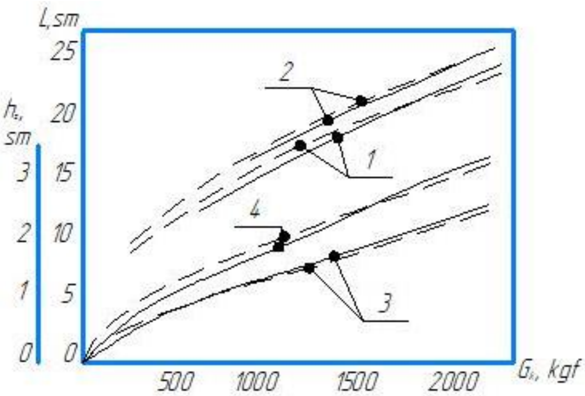

Fig. 4. **Dependences of the contact length L (curves 1, 2) and the normal deflection h_z (curves 3, 4) on the normal load O_k at an air pressure in a tire of 5.3 kgf·cm^{-2}, 1 and 3 are tires of size 260-20; 2 and 4 are tires of size 260-508P**

The equation for the radial displacements of the crown will have the form: for the contact area

$$w = -\frac{x^2}{2r^0} + \frac{L}{2r_0} + \frac{L}{8r_0} + w_0; \tag{18}$$

for the non-contact zone

$$w = -\frac{L\cosh\lambda_1(x - \pi r_0)}{2r^0\lambda_1\sinh\lambda_1\left(\frac{L}{2} - \pi r_0\right)} + w_0. \tag{19}$$

The analysis of the expression for determining the value shows that

$$\lambda_1 \approx \sqrt{\frac{N_0}{D}} \approx \sqrt{p_w r_1}, \tag{20}$$

where r_1 – proportionality coefficient.

As follows from equation (17), the parameter λ_1 determines the nature of the relationship between the contact length L and deflection h_z.

The parameter λ_1 is determined mainly by the ratio of the load on the crown of the tire in the circumferential direction under the action of internal pressure N_0 to the cylindrical stiffness of the crown of the tire D.

The parameter λ_1 is the determining one as follows from equations (18) and (19) [14] for radial displacements of the crown part of the tire.

Figures 3-6 show the experimental (solid lines) characteristics of tires of sizes 260-20, 260-508P and 6.00-13 and the calculation results (dashed lines) of the contact length and deflection of the tire at various values of normal load and internal air pressure, as well as for tires 260-20 radial movements w of the crown elements along the tire semicircle.

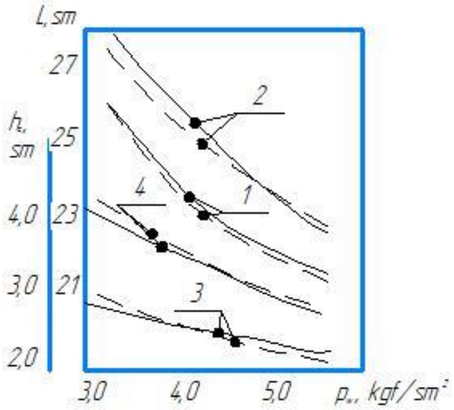

Fig. 5. **Dependence of the contact length *L* (curves 1, 2) and the normal deflection h_z (curves 3, 4) on the air pressure in the tire w at a normal wheel load of 1860 kgf**
(designations are the same as in Fig. 4)

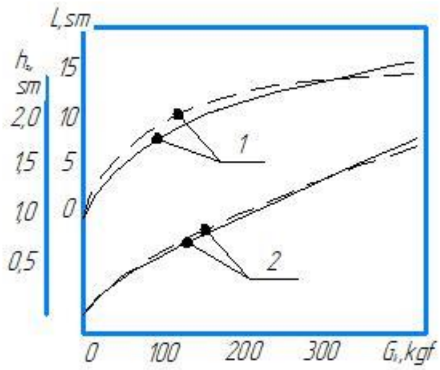

Fig. 6. **Dependence of the contact length *L* (curves 1) and the normal deflection h_z (curves 2) on the normal load G_k at an air pressure of 1.7 kgf·cm^{-2}. Tire 6.00-13**

During the calculation, the influence of rubber on the stiffness characteristics of the tire in equation (9) was neglected, which in this case, taking into account equitation (17), takes the form:

$$G_k = p_w b a_2 L \left(\frac{L}{2r^0 \lambda_1} + \frac{L^2}{12r^0} + w_0 \right). \tag{21}$$

The parameters of the tires, for which the value of w_0 can be neglected, are determined by the dependence

$$G_k = \frac{1}{2r_0} p_w b a_2 L^2 \left(\frac{1}{\lambda_1} + \frac{L}{6}\right). \tag{22}$$

Let us determine the length of the contact from the last equation

$$L = \frac{2}{\lambda_1}\left\{2\cosh\left[\frac{1}{3}\operatorname{arccosh}\left(\frac{3}{4}\frac{G_k r_0 \lambda_1^3}{p_w b a_2} - 1\right)\right] - 1\right\}. \tag{23}$$

The last three equations can be used to calculate tire parameters, which have a ratio of width to profile height of about one. The parameters of the other tires are determined by the initial equation (9), taking into account the geometric relationships. The initial data for the calculation are the design parameters of the tire, air pressure and normal load.

Using equations (17) and (21), we can find the coefficient of normal tire stiffness

$$C_z = \frac{dG_k}{dh_z} = p_w b a_2 \frac{4L + \lambda_1 L^3 + 4 r_0 \lambda_1 w_0}{2 + \lambda_1 L}. \tag{24}$$

To obtain the value of the coefficient of normal stiffness explicitly as a function of vertical weight, you should substitute the formula for L in the last equation, however, such an expression will be cumbersome and one must limit to the system of equations (17), (21) and (24). Calculating the coefficient of normal stiffness using this system is not difficult.

Conclusions

The following loads act on the element: tensile force N, bending moment M, shear force Q, force from the sidewall threads T, projections of force T on the x and z axes, respectively, shear forces Pc and Qc acting on the side faces of the element due to deformation of the rubber sidewalls, bending moment caused by deformation of the sidewall rubber Mc, normal and longitudinal tangential stresses acting from the tread rubber on the breaker and frame q and τ.

Under the influence of a normal load and the resulting displacement of the elements of the crown part of the tire w, the specific force N decreases in the circumferential direction, which corresponds to the experimental data.

The values of L, h_z, and C_z under equal loading conditions can vary by up to 10% for tires of the same model manufactured at different times. Given this, as well as the difficulty of calculating the values of the coefficient a_2 and parameter λ_1 (more precisely, r_1), in some cases it is advisable to determine them by introducing into equations derived in this paper one experimental value of the contact length L and one value of the normal deflection h_z of the tire under study, which was done in this work.

Author contributions

All authors have contributed equally to the study and preparation of this publication. Authors have read and agreed to the published version of the manuscript.

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