TRANSLATIONAL VIBRATIONS OF FLAT PLATE WITH ELASTIC SUSPENSION IN AIR FLOW

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Abstract. The goal of the study was exploring the possibility of energy harvesting from the vibration of the body in the air focusing on the example of the flat plate. Theory suggests that at certain conditions and excitation parameters vibrations of the system increase so that energy can be produced. Additional energy can be extracted from flutter motion – an aeroelastic instability that results in self-excited oscillation. The flutter is usually considered as the negative side effect that engineers try to eliminate, but in this study, we look for ways how to benefit from it. The flat plate is fixed in a flexible suspension and is affected by the air flow. The suspension is designed in such a way that the plate does not rotate but moves in a translational motion. First, the air flow was set up in different ways to study the response of the plate. Using computer modelling in MathCAD software, the movement of the object in the air flow was analysed. Some areas of the system at specific excitation conditions produced stable "flutter" type oscillations. Normally, the damping should be negative to describe self-excited vibrations. However, modelling proved that at specific lift and drag values the amplitude of the vibration grows even if there is no damping component in the equation at all. Graphs with changes in the center coordinate and velocity components of the 2DOF plate were composed to illustrate the solution.

Keywords: flutter; translational vibrations; energy extraction.

Introduction

Vibrations induced by the air flow are generally dangerous for structures and aircraft [1], but nowadays researchers are looking for possibilities to use specific vibration for human advantage. Flow induced vibrations should be handled with care, because under negative damping the system becomes unstable [1; 2]. Self-excited vibrations and Kármán vortices could negatively affect ship propellers, Francis turbines, submarine periscopes [2] and could lead to damage of the smokestacks (due to Kármán vortices resulting from alternate lateral forces), damage to transmission lines [1] due to galloping. Thus, we would like to compare flow induced vibrations to the high voltage electricity, because electricity has a killing force if handled improperly, but it could be as well managed to our benefit.

Vibrations in the air flow are often associated with the flutter phenomena. However, the flutter of the thin plate may be as well the result of magnetic field acting on a plate as in the example of hydropower generator stator core. Flutter in the air flow is a specific case of resonance, which means that there are two matching operational frequencies and resulting growth of vibration amplitude. The flutter of the airfoil associated with the resonance may occur when wind is continuously adding the energy to the system, and thus the natural frequency of the transverse vibration matches the Kármán vortex shedding frequency or when frequency of the torsional vibration of the structure matches one of the transverse vibration frequencies.

Researchers from the Al Azhar University described flutter as motion of two degrees of freedom (DOF), bending and torsion [3], and used Lagrange equation for mathematical modelling [3]. Indeed, in the theory of aircraft structures flutter is explained as an interaction of flexural and torsional motion [4]. Specifically, it is a type of resonance when the first torsion mode of the structure results in such angle of attack that lift forces start acting in phase with flapwise bending motion and are not compensated by structural damping [5]. Nevertheless, in both situations the damping of the system determines whether vibration would calm down or amplify.

Elastic suspension

According to the theory of self-excited vibrations, the alternating force that sustains the motion is created or controlled by the motion itself, and when the motion stops, the alternating force disappears [2]. It means that elastic suspension could be modified to prevent air flow vortex shedding or enhance it. When the frequency of the vortexes resulting from the air flow no longer matches some of the vibration modes of the flat plate, the self-excited vibrations would stop.

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Forces acting on the system

Flow-induced vibrations, flatter and Kármán vortices may be presented as forced vibration problems [1] or self-excited vibration [2]. Let us present the assessment of the forces acting on the airfoil – the gravity force, lift force, which counteract one another in a vertical plane, drag force and propulsion force (if any propulsion is added to the system). External forces acting on the system may not necessarily be harmonic, wind may as well be a steady force, but it should continuously add energy to the system [1], as it happened in the history during the Tacoma bridge disaster [6]. D. Anderson in his book [1] provides the example of the flatter of the flag. For the flag, the propulsive force is generated by the wind. On the other hand, this force can be described as a negative damping – as an air resistance, which enhances motion instead of damping it. In this example, the gravity acts in vertical direction. We can imagine that the flag without a wind (no propulsion force) is going to hang limply down from the pole. Lift counteracts the gravity force when the wind is present.

Negative damping

Since the system itself defines the capacity for self-excited vibrations, not the sustaining alternating force as in the case of the forced vibration, a self-excited vibration is described as a free vibration with negative damping [2]. We used this concept for the mathematical model used for the calculations, as shown in equation (1) in Materials and methods section. Using negative damping for modelling is convenient because wind causing the flutter could not be described by the constant force. The constant force does not have a period and cannot be resonant with the period of the structure. To generate resonant vortices, there should be a steady aerodynamic force. In this paper, we will first consider the motion of 1 DOF system from the effect of the air flow, when in the mathematical model the effect of the second degree of freedom is replaced by negative friction. We will then consider a more complex model of 2 DOF of translational motion, in which the centre of mass of a plate in a flatter motion forms an ellipse-like trajectory.

Materials and methods

In our previous studies models with three DOF were used [7]. For this study a mathematical model of an airflow (flat plate) suspended in steady flow was used with one DOF (Fig. 1.) with either positive or negative damping and two DOF (Fig. 2.) without damping:

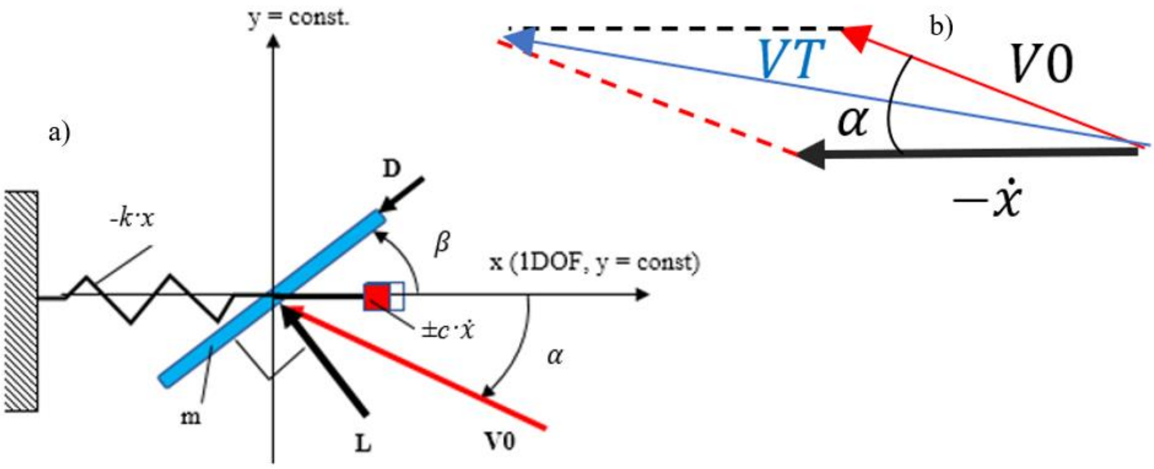

Fig. 1. **Free body diagram (a) and vector diagram (b) of the flat plate with 1 DOF,** L – lift force; D – drag force; m – mass; c – damping; k – stiffness of the spring supporting the plate; α – angle of air flow attack; x – displacement of the plate; β – position of the plate with respect to x axis; $V0$ – air flow velocity; VT– total velocity

According to Newton's second law of mechanics, the differential equation of motion in a 1DOF system with coordinate x will be as follows (1), where the third row of equation (1) is a combination of the equations used for mathematical model, rewritten in the form suitable for the explicit Euler step integration method:

$$\begin{cases} m\ddot{x} - c\dot{x} + kx = 0 \\ -Lsin\alpha - Dcos\alpha - kx = m\ddot{x} \end{cases} \Rightarrow$$
$$\frac{dv}{dt} = \frac{1}{m}(-Lsin\alpha - Dcos\alpha + (-c)\dot{x} - kx), \quad (1)$$

where m – mass of the plate, kg, chosen to be here 1 kg for the simulation;
$\ddot{x}, \dot{x}, x$ – horizontal acceleration, velocity and displacement of the plate, m·s^{-2}, m·s^{-1}, m;
c – coefficient of viscous damping of the system, positive or negative, N·s·m^{-1};
k – stiffness of the spring supporting the plate, here 981 N·m^{-1};
L – lift force, N;
α – angle of air flow attack, º;
D – drag force, N;
$\frac{dv}{dt}$ – vertical acceleration of the plate, m·s^{-2}.

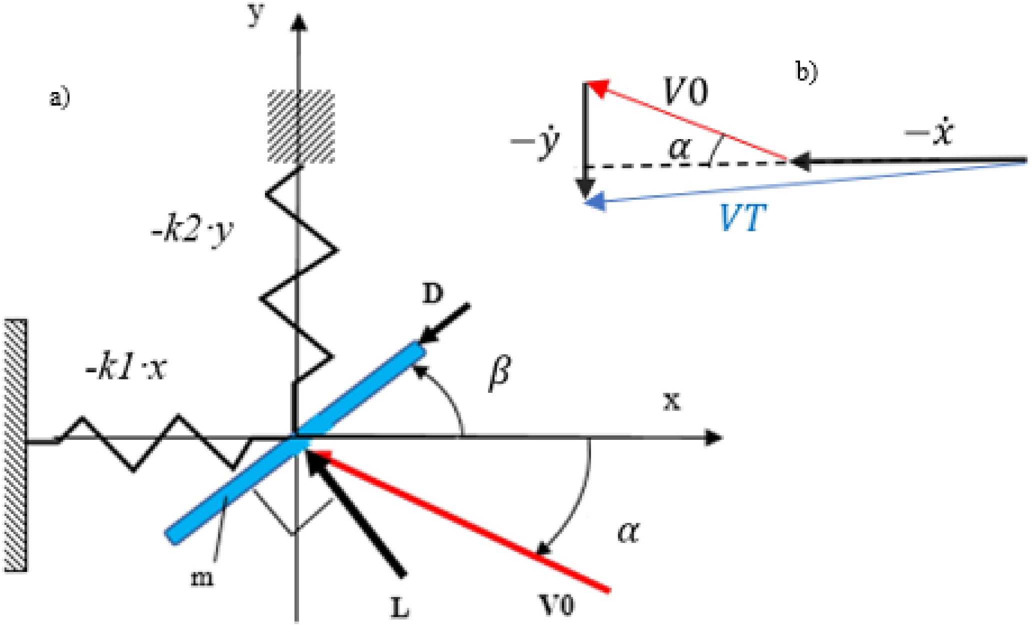

Fig. 2. **Free body diagram of the flat plate (a) and vector diagram (b) of the flat plate with 2 DOF and no damping,** $k1, k2$ – stiffness of the spring supporting the plate

For the model with 2 DOF and no damping the equations become as shown in equation (2):

$$\begin{cases} m\ddot{x} = -k1 \cdot x - Lsin\alpha - Dcos\alpha; \\ m\ddot{y} = -k2 \cdot y + Lcos\alpha - Dsin\alpha, \end{cases} \quad (2)$$

where $\ddot{x}, \ddot{y}, x, y$ – horizontal and vertical acceleration, and displacement, m·s^{-2}, m;
$k1, k2$ – stiffness of the spring supporting the plate, here 981 N·m^{-1}.

The gravity force and lift force counteract in a vertical plane, y, the drag force acts in the x plane. In our previous paper [8] we proposed to enrich the drag and lift force equations from fluid mechanics fundamentals on the external flow [9] with the velocity of the air, because recent studies showed that it has an effect on drag and it is a parabolic function [10]), and the signum function of the total velocity, as proposed in the previous papers [11; 12], as shown in the resulting equation (3):

$$D = \frac{1}{2}\rho C_D A(VT_x)^2 \cdot sign(VT_x);$$
$$L = \frac{1}{2}\rho C_L S(VT_y)^2 \cdot sign(VT_y), \quad (3)$$

where D, L – drag force and lift force, N;
ρ – fluid density, 1.25 kg·m^{-3};
C_D, C_L– drag coefficient, lift coefficient, depend on the shape of the object, unitless, previously obtained using simulation in paper [11];

A, S – area of the plate that force is acting on, m^2, for the simulation the sides of the plates were chosen to be 1 m long and the thickness of the plate was 0.01 m;
VT_x, VT_y – total velocity of the plate and the air flow, m·s^{-1}.

Results

The simulation results confirmed that the damping value has a critical effect on the growth of the vibration amplitude. The selected results are presented below in Fig. 3 for the negative coefficient of viscous damping value –5 N·s·m^{-1}, lift coefficient 0.5, drag coefficient 0.05, angle of attack 0º:

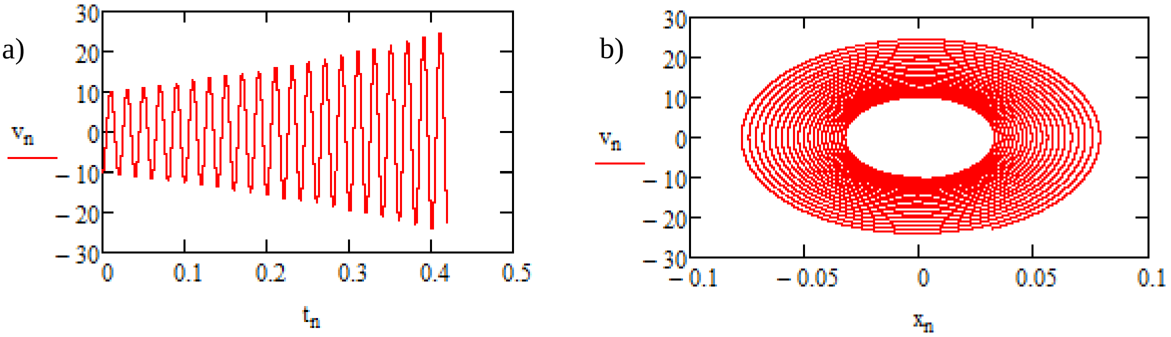

Fig. 3. **1 DOF flatter motion in wind flow $V0$ = 10 m·s^{-1} for the plate oriented at the angle $\beta = \pi/6$º with respect to horizontal axis:** a – velocity v as time t function; b – motion in phase plane

For the following simulation the number of simulation steps had to be adjusted for the angle of attack greater than 60º, namely, it was 60 000 instead of 200 000. Simulation step size was adjusted as well, when required by code. In this simulation we would like to highlight the importance of the lift and drag force on the amount of vibration produced, therefore we added the comments about maximum and minimum lift and drag forces according to Figure 12.4 of the book [1] in Fig.4 below:

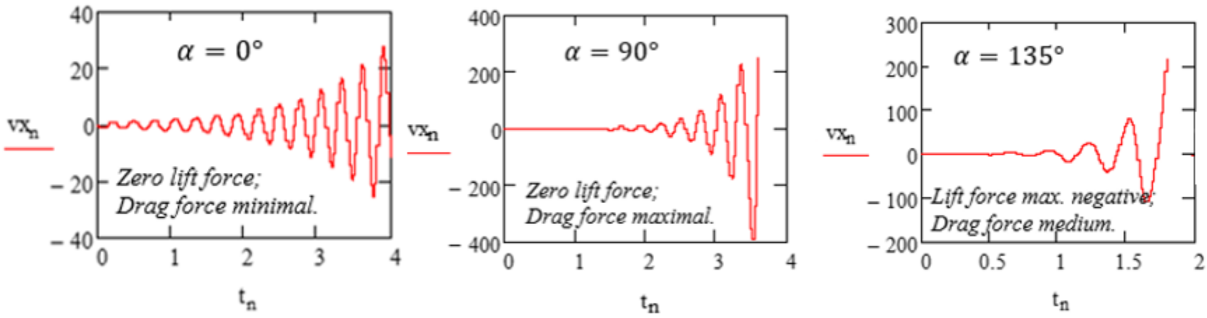

Fig. 4. **Vibration velocity amplitude increase given negative damping depending on the angle of attack:** α varying from zero to 180º

Comparing the plots for zero and 90º, where the lift force is zero, one can evaluate the effect of the drag force on the total vibration velocity amplitude. When the drag force is small, the amplitude can grow up to 20 m·s^{-1} in 4 seconds, but when the drag force is maximum, it grows even more, but it is only due to negative damping. When the negative damping is removed from the equation, the amplitude of vibrations at the angle of attack equal to 90º is on the contrary decreasing.

The instability of the system with a negative lift force was explained in the literature before [2], but the difference in the amplitude has not been simulated previously. We observed that under specific lift and drag values the amplitude of the vibration grew even if we deleted the damping component from the equation. The amplitude of the vibration velocity grew because the lift force was negative, for example, as shown in Fig. 5.

Following the book [1], Figure 12.4., this effect of the lift force starts acting at the angle of attack values greater than 90º, but at 100º the effect is not yet visible, because the drag force is still close to the maximum. The negative slope of the lift curve should be greater that the ordinate of the drag curve [2] to achieve instability. Thus, when the drag force decreases, starting at around 115º, the effect of negative lift takes place. This system is unstable. It is understood that this is only possible, when the wind is blowing with some initial velocity, for the amplitude simulation above it was chosen to be 10 m·s^{-1} to observe the differences better. The amplitude grew 10 times comparing the angle of attack 100° and 135°. The results of numerical modeling for 2DOF system are shown in Fig. 6 for the angle of attack $\pi/3°$ and orientation of the plate with respect to the horizontal axis $\pi/6°$ (Fig. 6).

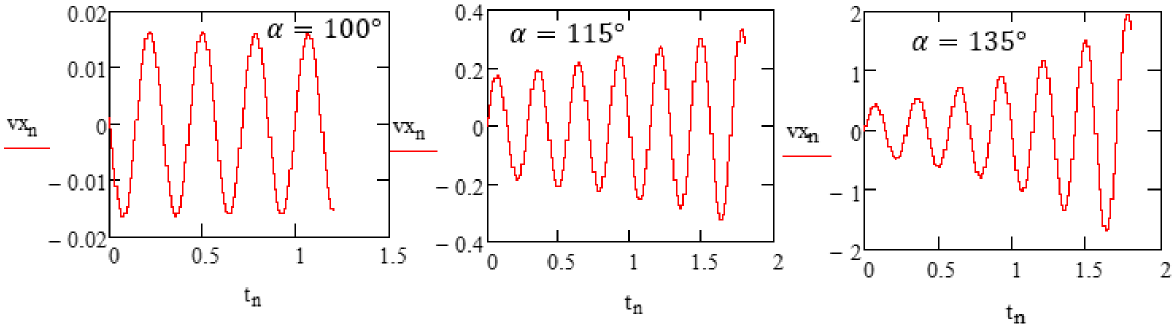

Fig. 5. **Equation with no damping; the amplitude is growing because the system is unstable when the lift force has specific negative values**

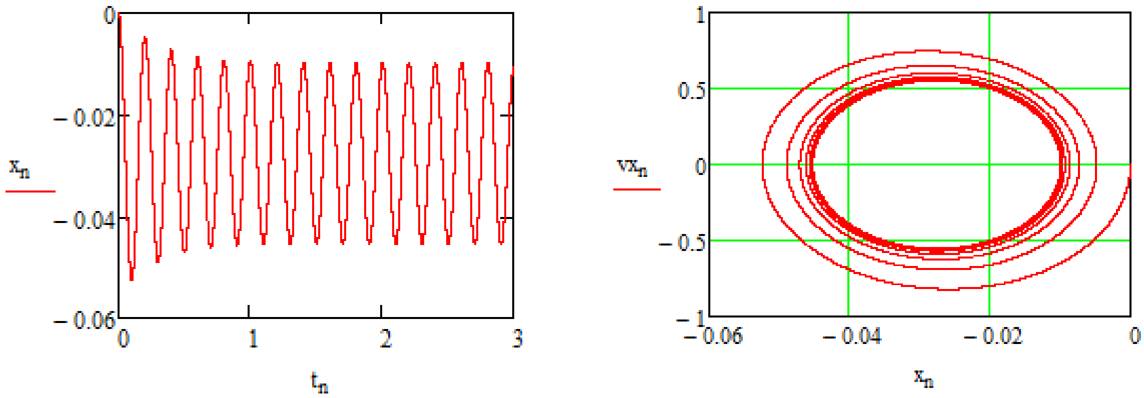

Fig. 6. **2 DOF flatter motion in wind flow:**
a – velocity *v* as time *t* function; b – motion in phase plane

From Fig.6 b) it can be concluded that the transition process has started for the x coordinate from outside the boundary cycle and for the y coordinate from inside the boundary cycle. In general, it was shown here that flaters occur in a system of 2 DOF without using any additional assumptions (negative damping, additional rotation).

Discussion

1. The occurrence of flatter vibrations in a constant air flow in a system of 1 DOF can be explained by generation of energy from negative friction, which approximates the unobserved additional motion: from the rotational or translational motion of the plate.
2. The main disadvantage of the 1 DOF system model is that some system or excitation parameters have to be assumed in advance at the beginning of the analysis. They should then be tested experimentally or with an additional 2 DOF model.
3. The model of 2 DOF allows to study the origin of flatter motion without using any assumptions, except for drag and lift coefficients.
4. Numerical analysis shows that the oscillation motion in both components occurs at the same frequency, only with a small phase shift.
5. 5. Analysis of 2 DOF translational motions shows that the flatter is a typical phenomenon of nonlinear systems.

Conclusions

1. The damping should be negative to describe self-excited vibrations.
2. Under specific lift and drag values the amplitude of the vibration grows even if there is no damping.
3. The effect of the instability starts acting at special angles of plane positioning, also considering the angle of the air flow.

4. The developed methodology of flat plate motion analysis of 2DOF system can be applied in motion analysis, parameter optimization and synthesis of new systems, applying it in the prevention of oscillations or in the extraction of energy from the air flow.

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