MECHANISM MOTION STUDIES WITH COLLISIONS AT SEVERAL POINTS

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Abstract. The paper analyses the motion of mechanical systems, which causes simultaneous collisions (shocks) in their elements at more than two contact points. The main hypothesis, which is used in the theoretical calculations, is that the mentioned impacts begin and end at the same time at all impact contact points. Impact interaction of contact points includes normal reaction and dry friction force with correlations under stopping areas. Description of the relationship impacts at the same time at two, three or more points is given. For the description of the plane motion for mechanical systems the mass center motion theorem and the theorem of kinetic momentum changes are used. Acquired correlations are used in the mechanical systems plane motion analysis of transition and stationary motion regimes under a number of collision points. The modelling with MathCAD program is carried out for systems with one, two and three degrees of freedom, which takes place in collisions in two and more points. The inspection of theory has been performed by the Working Model 2D Program for mechanical systems with one, two, three degrees of freedom with collisions at three or more points. Validation of studies is significant. In addition the demonstration results for experimental studies for vibro drive system are given. Accordingly, the vibro engine is created by disbalance motors of the pendulum platform in plane motion. Impacts are generated in two flat springs after rotation abruption from fundament. The results of the work may be used in calculations of equipment in the mechanical positioning system, as well as in the design of new mechanisms or machines, such as vibro conveyors or vibro engines.

Keywords: shocks, impacts in mechanical system, simultaneous collision modelling.

Introduction

Example of collision interaction in one point can be described by normal reaction N and dry friction force $\pm F$ (Fig. 1). Graphics of normal reaction and one dry friction force (for full slip $+F$) in time domain is shown in Fig. 2.

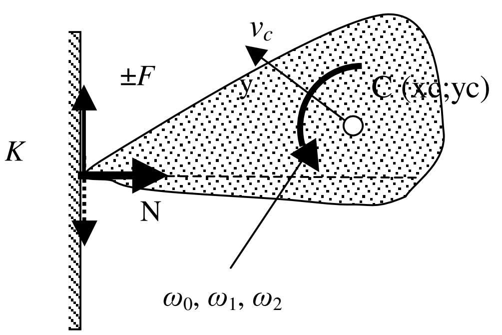

Fig. 1. **Model of collisions in point *K*:** *N* – normal reaction; $\pm F$ – dry friction force with variable direction (±) along tangent in contact point *K*, ω_0, ω_1, ω_2 – angular velcity of body in three time moments (initial, in the middle, at the end)

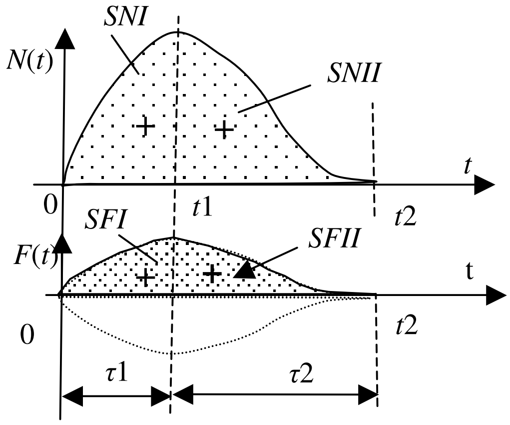

Fig. 2. **Graphics of normal reaction and dry friction force (for full slip +*F*) in time domain:** $\tau 1$ – time interval of the first phase of collision; $\tau 2$ – time interval of the second phase of collision

Here $F(t) = f \cdot N(t)$;

$$SNI = \int_0^{\tau 1} N(t) \cdot dt; \quad SNII = \int_{\tau 1}^{\tau 1+\tau 2} N(t) \cdot dt; \quad SFI = \int_0^{\tau 1} F(t) \cdot dt; \quad SFII = \int_{\tau 1}^{\tau 1+\tau 2} F(t) \cdot dt, \qquad (1)$$

where f – dry friction coefficient;
SNI, *SNII* – impulses of normal reaction *N*(*t*);

SFI, *SFII* – impulses of dry friction force *F*(*t*) in the first and second impact phase.

According to the theory of body plane collisions with obstacle in one point, here for collisions at several points the existence of one of seven different impact interaction cases can be checked as follows [1-4].

1. Full slips depending on the initial contact point *K* sufficient tangential velocity component.
2. Full slip in one direction depending of special body geometry and with special motion initial condition (when the contact point *K* has zero tangential velocity component).
3. Partial slip with end at the first phase of impact ($t \leq \tau 1$).
4. Partial slip with end at the second phase of impact ($\tau 1 \leq t \leq \tau 1 + \tau 2$).
5. Full slip in two directions with dry friction force reverse in the first phase of impact ($t \leq \tau 1$).
6. Full slip in two directions with dry friction force reverse in the second phase of impact ($\tau 1 \leq t \leq \tau 1 + \tau 2$).
7. No slip in the contact point.

Four of these seven interaction cases are shown in Fig. 2-4.

These seven different impact interaction cases [5] can be used for the investigation of collisions at several points [4].

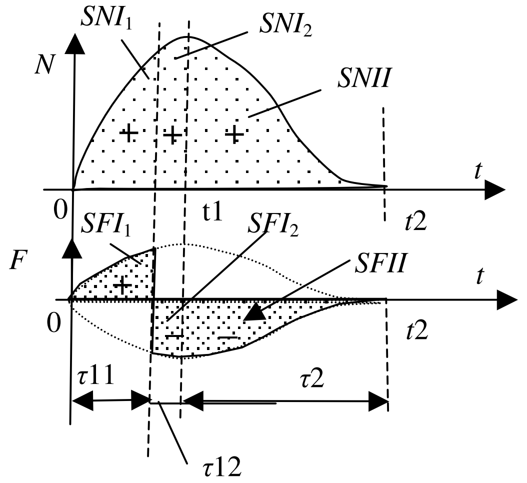

Fig. 3. **Graphic of normal reaction and dry friction force (full slip in two directions) in a case 5:** SFI_1 – dry friction force impulse before reverse in the first phase of impact ($\tau 11 \leq \tau 1$); SFI_2 – negative dry friction force impulse after reverse till $\tau 1$; SNI_1, SNI_2 – parts of normal reaction impulse in the first phase of impact

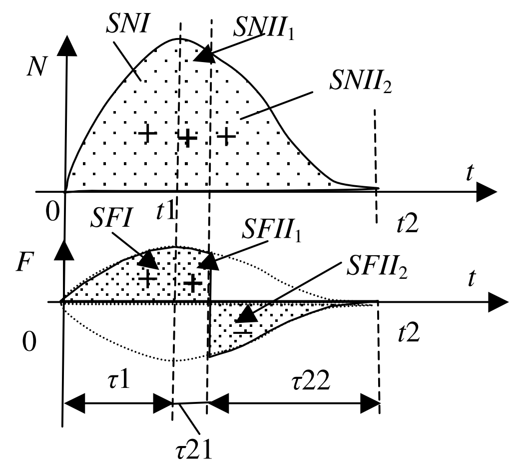

Fig. 4. **Graphic of normal reaction and dry friction force (full slip in two directions) in a case 6:** $SFII_1$ – dry friction force impulse before reverse in the second phase of impact ($\tau 1 + \tau 21 \leq t \leq \tau 1 + \tau 2$); $SFII_2$ – negative dry friction force impulse after reverse till $\tau 1 + \tau 2$; $SNII_1$, $SNII_2$ – parts of normal reaction impulse in the second phase of impact

Simultaneous collisions of rotating body in two points

Collisions in two points in the body with one degree of freedom will be observed. Assume that the rotating body collides with a rigid obstacle in the point *K* (Fig. 5). According to the collision theory the weight and other large distance interactions can be neglected [1-6]. Then impact forces will be: in the axis *O* forces with components *XO*, *YO*; reactions in the contact point *K* like normal reaction *N* and dry friction force *F*. The idea of calculation includes a possibility to write equations of mechanics in special time intervals: in the time moment when the contact point *K* velocity normal component is zero; the time moment when the contact point *K* tangential velocity component is changing direction (e.g., dry friction force changes direction, too). In a case of full slipping at the contact point *K* (when after collisions the body springs at an obstacle) normal reaction *N*(*t*) and dry friction force *F*(*t*) in time t domain are shown in Fig. 6.

For the given model using classical mechanics equations about exchange of linear momentum for center mass *C* and angular momentum against the rotation axis *O* in time moments *τ*1 and *τ*2 can be written formulas (2-4), [3; 6; 7]:

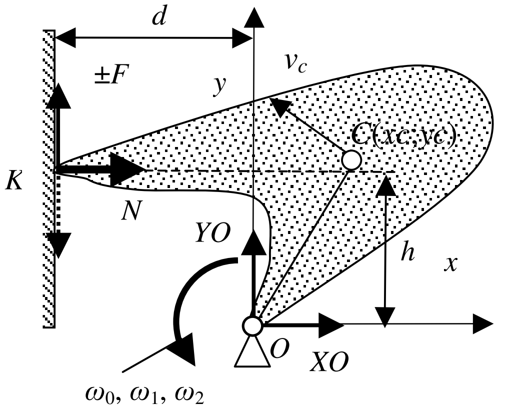

Fig. 5. **Model of collision of rotating body**

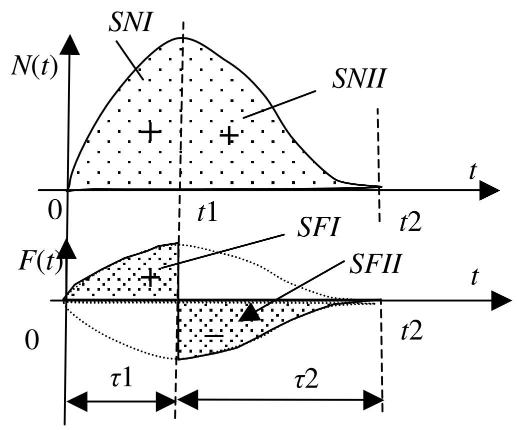

Fig. 6. **Diagrams of normal reaction *N*(*t*) and dry friction forces *F*(*t*) in time *t* domain**

$$0-(-m\cdot\omega 0\cdot yc)=SXI+SNI;\quad 0-m\cdot\omega 0\cdot xc=SYI+SFI;$$
$$0-JO\cdot\omega 0=-SNI\cdot h-SFI\cdot d; \tag{2}$$

$$-m\cdot\omega 2\cdot yc-0=SXII+SNII;\quad m\cdot\omega 2\cdot xc-0=SYII-SFII;$$
$$JO\cdot\omega 2-0=-SNII\cdot h+SFII\cdot d; \tag{3}$$

$$SNII=R\cdot SNI;\quad SFI=f\cdot SNI;\quad SFII=f\cdot SNII, \tag{4}$$

where *m* – mass of the body;
JO – moment inertia body against perpendicular axis in point *O*;
xc, *yc* – coordinates of center mass;
*ω*0 – initial angular velocity of the body;
R – coefficient of normal impulse restitution;
f – dry friction coefficient;
SNI, *SNII*, *SFI*, *SFII* – impulses of normal reaction and dry friction force in the first and second impact intervals τ1 and τ2;
SXI, *SXII*, *SYI*, *SYII* – impulses of reaction force components *XO* and *YO* at point *O* (Fig. 5).

Eight unknowns can be found from equations (2) – (4):

$$\omega 2=\frac{R\cdot\omega 0\cdot(d\cdot f-h)}{h+f\cdot d};\quad SNI=\frac{JO\cdot\omega 0}{h+f\cdot d};\quad SNII=\frac{R\cdot JO\cdot\omega 0}{h+f\cdot d};$$
$$SFI=\frac{f\cdot JO\cdot\omega 0}{h+f\cdot d};\quad SFII=\frac{R\cdot f\cdot JO\cdot\omega 0}{h+f\cdot d}; \tag{5}$$

$$SXI=\frac{(h+f\cdot d)\cdot m\cdot\omega 0\cdot yc-JO\cdot\omega 0}{h+f\cdot d};SXII=R\cdot\frac{(h-f\cdot d)\cdot m\cdot\omega 0\cdot yc-JO\cdot\omega 0}{h+f\cdot d};$$
$$SYI=-\frac{(h+f\cdot d)\cdot m\cdot\omega 0\cdot xc+f\cdot JO\cdot\omega 0}{h+f\cdot d}; \tag{6}$$
$$SYII=R\cdot\frac{(-h+f\cdot d)\cdot m\cdot\omega 0\cdot xc+f\cdot JO\cdot\omega 0}{h+f\cdot d}.$$

Generally known result about jamming in dry friction mechanisms follows from the first formula (3), e.g.:

if $h > f \cdot d$, then $\omega 2 < 0$ and the body springs back from the obstacle. Otherwise, if $h < f \cdot d$ jamming will be and the body will stick to the obstacle.

If additionally the time of collision $\tau 1 + \tau 2$ is given, middle values of impact forces *N*, *F*, *XO*, *YO* (Fig. 2, 5, 6.) can be calculated from formulas (2-4).

For optimizations of the system parameters *h*, *d*, *xc*, *yc*, *m* and *JO* when a criterion is given (for example, minimum of impact impulses in rotating axe) formula (6) can be used.

Simultaneous collisions of rotating body in four points

The model of collision is shown in Fig. 7, where such collision configuration is investigated, in which all impact points are placed in one line *O*, *K*1, *K*2 and *K*3. In these points there are not dry friction interactions, because velocities of all contact points are perpendicular to the obstacle.

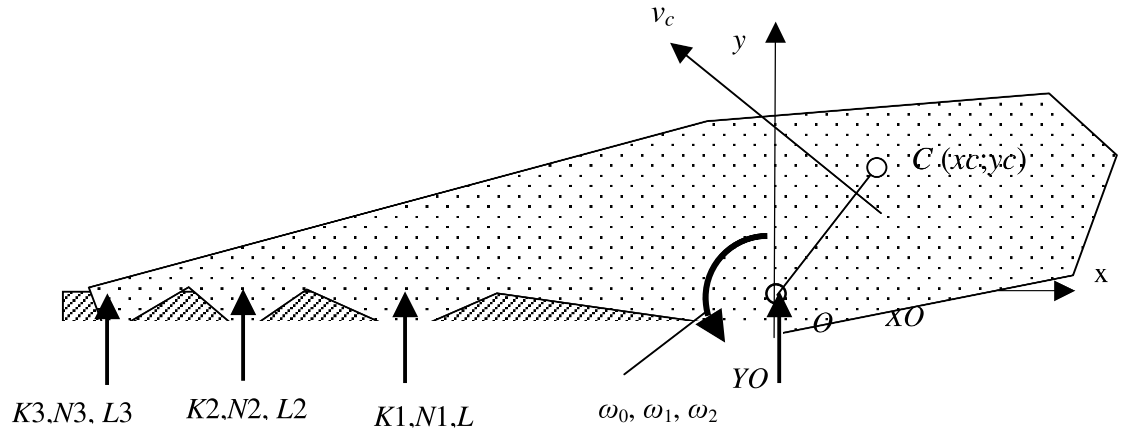

Fig. 7. **Model of simultaneous collisions of rotating body in four points**

For a rotating body when collisions take place simultaneously in four points (three points K1, K2, K3 of the obstacle and one in the axis O) the above described theory and recommendations are used.

For rotating motion at the end of the first impact phase and at the end of full impact can be found [4,6]:

$$\begin{aligned} &0 - JO \cdot \omega 0 = -S1I \cdot L1 - S2I \cdot L2 - S3I \cdot L3; \\ &JO \cdot \omega 2 - 0 = -R1 \cdot S1I \cdot L1 - R2 \cdot S2I \cdot L2 - R3 \cdot S3I \cdot L3, \end{aligned} \tag{7}$$

where *R*1, *R*2 and *R*3 – coefficients of restitutions in different points *K*1, *K*2 and *K*3;
*L*1, *L*2 and *L*3 – distances from the axe O till points *K*1, *K*2 and *K*3;
*S*1*I*, *S*2*I*, *S*3*I* – normal impulses at the end of the first phase.

For motion of center *C* mass can be calculated:

$$\begin{aligned} &m \cdot \omega 2 \cdot yc - m \cdot \omega 0 \cdot yc = SXO; \\ &m \cdot \omega 2 \cdot xc - m \cdot \omega 0 \cdot xc = (1 + R1) \cdot S1I + (1 + R2) \cdot S2I + (1 + R3) \cdot S3I + SYO, \end{aligned} \tag{8}$$

where SXO, SYO – reaction impulse components in the axis O;
xc, yc – coordinates of center mass C.

According to equations of constraint proportionality can be used:

$$S1I = S3I \cdot \frac{L1}{L3}; \quad S2I = S3I \cdot \frac{L2}{L3}. \tag{9}$$

From equations (7-9) all six unknowns *ω*2, *SXO*, *SYO*, *S*1*I*, *S*2*I*, *and S*3*I* can be found. For example:

$$\omega 2 = \frac{R1 \cdot \omega 0 \cdot L1^2 + R2 \cdot \omega 0 \cdot L2^2 + R3 \cdot \omega 0 \cdot L3^2}{L1^2 + L2^2 + L3^2}.$$

For optimization of the system parameters formulas (7-9) can be used when a criterion and limits are given.

Simultaneous collisions in vibro driver system

The vibro impact driver model for horizontal motion is shown in Fig. 8. The system is excited by the rotating eccentric 5 when the actuator 4 starts to draw away from the main collision element. This time moment impacts in contact points *K*1-*K*4 occur. Using the theory above, the system parameters for motion to the right side was found. This complicated mechanism was investigated by Working Model – 2D program. An example of platform velocity obtained from modelling is shown in Fig. 9. According to the given results a real driver was made (Fig. 10).

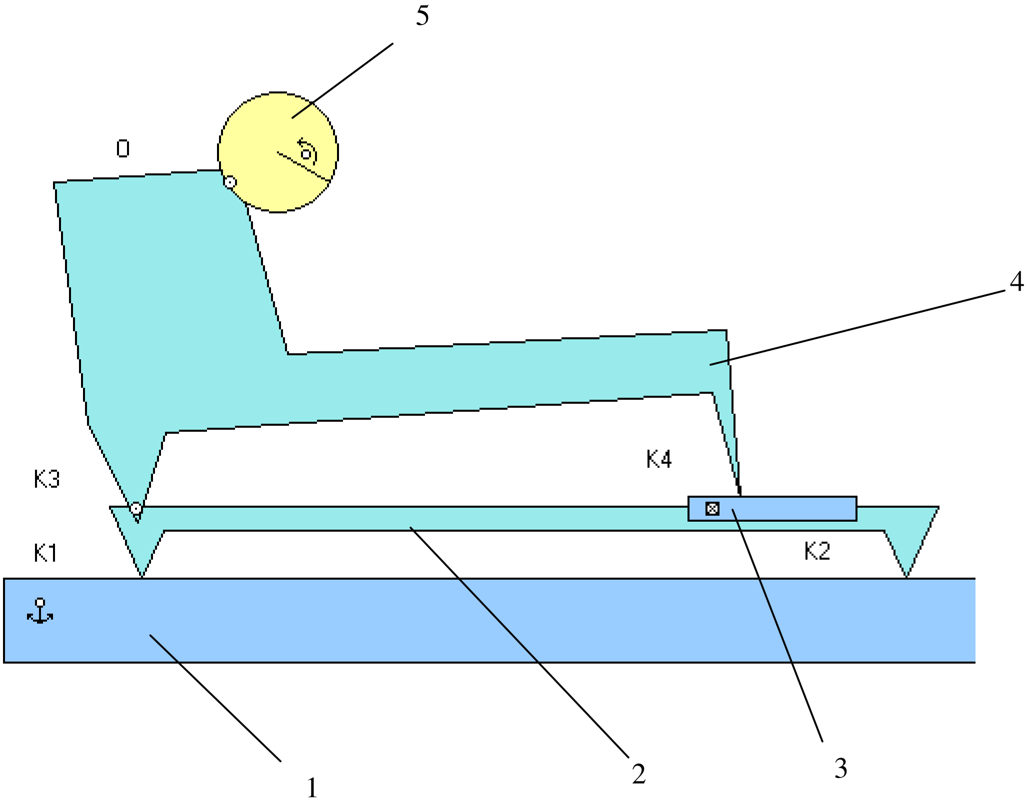

Fig. 8. **Horizontal vibro driver system:** 1 – fundament; 2 – moving platform; 3 – main collision element; 4 – vibro impact actuator; 5 – eccentric, which is driven by electro motor; *K*1; *K*2; *K*3 and *K*4 – impacts points; *O* – rotation axis of the eccentric

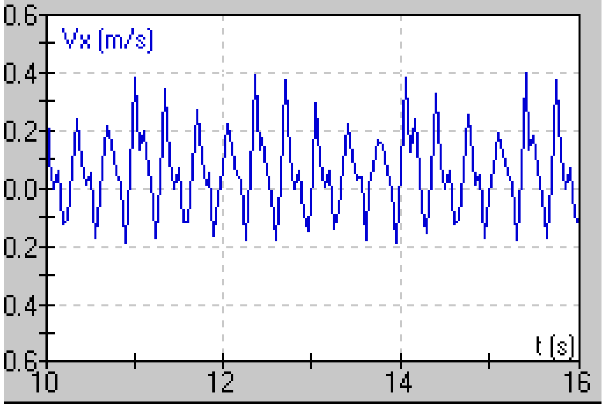

Fig. 9. **Horizontal velocity graphics for moving platform 2**

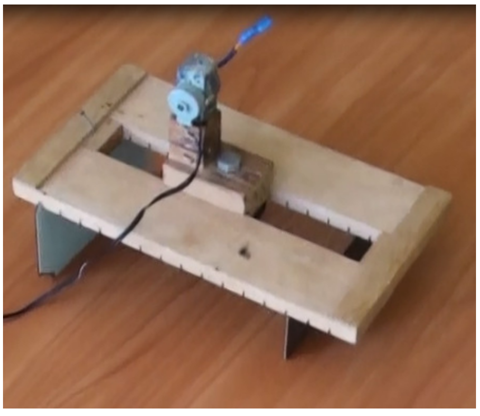

Fig. 10. **Experimental model with eccentric**

Results and discussion

1. It is possible to calculate simultaneous impacts at several points using coefficients of restitutions and dry friction coefficients.
2. Given calculation method allows to find analytical formulas for simultaneous collisions at several points.
3. Analytical formulas must be used for the system parameter optimization.
4. Results of modelling with Working Model were used for design of the experimental model.

Conclusions

1. Here given method for calculation of impacts at several points is applicable to shock calculations, mostly for one solid body: in plane or rotation motions by means of the classical mechanics relationships.
2. For calculations of systems with impacts at more than three points computer programs can be used [7].
3. The analysis of impacts in complicated systems by computer programs needs checking of additional results, which takes into account seven different impact cases [7].

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