KINEMATIC SYNTHESIS OF INITIAL KINEMATIC CHAINS (IKC)

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Abstract. A solution to the problem of synthesizing an initial three-dimensional kinematic chain with spherical and rotary kinematic pairs is presented. It is shown that this chain can be used as a structural module for structural-kinematic synthesis of motion of three-dimensional four-link motion-generating lever mechanisms by the preset positions of the input-and output links.

Key words: mechanism, link, kinematic pairs, initial kinematic chains, synthesis.

Introduction

The work specifies the solution of the problem of synthesis of the original spatial kinematic chain with spherical and rotational kinematic pairs and shows its use as a structural module with structural kinematic synthesis of spatial four-link moving linkage mechanism as per specified positions of the input and output links [1].

Problem statement: given N of finite distant positions of two solids Q_1 and Q_2

$$Q_1(\theta_i^1, \psi_i^1, \phi_i^1) \;\; Q_2(X_{Di}, Y_{Di}, Z_{Di}, \theta_i^2, \psi_i^2, \phi_i^2) \;\; i = \overrightarrow{1,N} \;, \tag{1}$$

where θ_i^j, ψ_i^j, ϕ_i^j – fixed axis Eulerian angles $OXYZ$;

X_{Di}, Y_{Di}, Z_{Di} – the position of the point D_i of the solid Q_2.

It is required to find such points in the fixed axis as $A(X_A, Y_A, Z_A)$, $B(x_B, y_B, z_B)$ of the solid Q_1 and $C(x_C, y_C, z_C)$ of the solid Q_2, so that distance between the points B and C in all positions of the solids Q_1 and Q_2 is a little different from the constant value R (see Fig. 1).

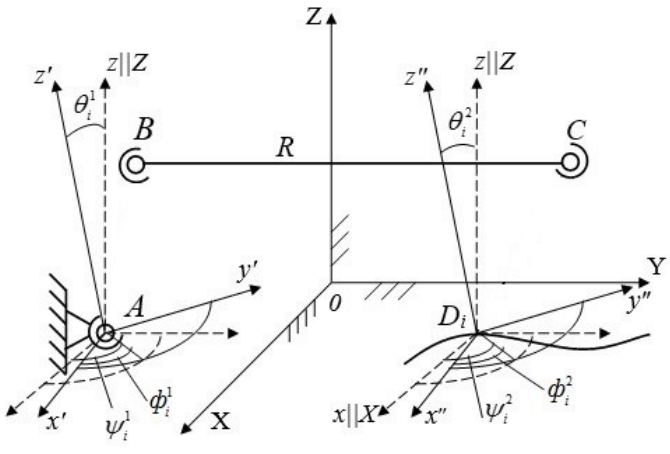

Fig. 1. **Equivalent four-link initial kinematic chain *ABCD***

Problem solution: Let us introduce the weighted difference for i position of solids in a form [2]

$$\Delta_{q_i} = \left|\overrightarrow{B_i C_i}\right|^2 - R^2 = (X_{C_i} - X_{B_i})^2 + (Y_{C_i} - Y_{B_i})^2 + (Z_{C_i} - Z_{B_i})^2 - R^2 \;\; i = \overrightarrow{1,N} \;, \tag{2}$$

where

$$\begin{bmatrix} X_{B_i} \\ Y_{B_i} \\ Z_{B_i} \\ 1 \end{bmatrix} = \begin{bmatrix} & [T_{10}^i] & & X_A \\ & & & Y_A \\ & & & Z_A \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_B \\ y_B \\ z_B \\ 1 \end{bmatrix}; \quad \begin{bmatrix} X_{C_i} \\ Y_{C_i} \\ Z_{C_i} \\ 1 \end{bmatrix} = \begin{bmatrix} & [T_{20}^i] & & X_{Di} \\ & & & Y_{Di} \\ & & & Z_{Di} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_C \\ y_C \\ z_C \\ 1 \end{bmatrix}$$

and

$$T^{i}_{jo} = \begin{bmatrix} e^{j}_{i1} & e^{j}_{i2} & e^{j}_{i3} \\ m^{j}_{i1} & m^{j}_{i2} & m^{j}_{i3} \\ n^{j}_{i1} & n^{j}_{i2} & n^{j}_{i3} \end{bmatrix} \quad \begin{matrix} j = \overrightarrow{1,2} \\ i = \overrightarrow{1,N} \end{matrix}, \tag{3}$$

where

$$\begin{cases} e^{j}_{i1} = \cos\psi^{j}_{i}\cdot\cos\phi^{j}_{i} - \cos\theta^{j}_{i}\cdot\sin\psi^{j}_{i}\cdot\sin\phi^{j}_{i} \\ m_{i1} = \sin\psi^{j}_{i}\cdot\cos\phi^{j}_{i} + \cos\theta^{j}_{i}\cdot\cos\psi^{j}_{i}\cdot\cos\phi^{j}_{i} \\ n^{j}_{i1} = \sin\theta^{j}_{i}\cdot\sin\phi^{j}_{i} \\ e^{j}_{i2} = -\cos\psi^{j}_{i}\cdot\sin\phi^{j}_{i} - \cos\theta^{j}_{i}\cdot\sin\psi^{j}_{i}\cdot\cos\phi^{j}_{i} \\ m^{j}_{i2} = -\sin\psi^{j}_{i}\cdot\sin\phi^{j}_{i} + \cos\theta^{j}_{i}\cdot\cos\psi^{j}_{i}\cdot\sin\phi^{j}_{i} \\ n^{j}_{i2} = \sin\theta^{j}_{i}\cdot\cos\phi^{j}_{i} \\ e^{j}_{i3} = \sin\theta^{j}_{i}\cdot\sin\psi^{j}_{i} \\ m^{j}_{i3} = -\sin\theta^{j}_{i}\cdot\cos\psi^{j}_{i} \\ n^{j}_{i3} = \cos\phi^{j}_{i} \end{cases}. \tag{4}$$

It is a function of ten parameters: X_A, Y_A, Z_A, x_B, y_B, z_B, R, x_C, y_C, z_C. By grouping these parameters in fours with the common parameter R, let us represent the weighted difference in three different forms

$$\Delta^{(1)}_{q_i} = (\tilde{X}_{A_i} - X_A)^2 + (\tilde{Y}_{A_i} - Y_A)^2 + (\tilde{Z}_{A_i} - Z_A)^2 - R^2, \tag{5}$$

$$\Delta^{(2)}_{q_i} = (\tilde{x}_{B_i} - x_B)^2 + (\tilde{y}_{B_i} - y_B)^2 + (\tilde{z}_{B_i} - Z_B)^2 - R^2, \tag{6}$$

$$\Delta^{(3)}_{q_i} = (\tilde{x}_{C_i} - x_C)^2 + (\tilde{y}_{C_i} - y_C)^2 + (\tilde{z}_{C_i} - Z_C)^2 - R^2, \tag{7}$$

where

$$\begin{bmatrix} \tilde{X}_{Ai} \\ \tilde{Y}''_{Ai} \\ \tilde{Z}_{Ai} \\ 1 \end{bmatrix} = \begin{bmatrix} & [T^{i}_{10}] & & \begin{matrix}0\\0\\0\end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_B \\ y_B \\ z_B \\ 1 \end{bmatrix} + \begin{bmatrix} & [T^{i}_{20}] & & \begin{matrix}X_{D_i}\\Y_{D_i}\\Z_{D_i}\end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_C \\ y_C \\ z_C \\ 1 \end{bmatrix} \tag{8}$$

$$\begin{bmatrix} \tilde{x}_{B_i} \\ \tilde{y}_{B_i} \\ \tilde{z}_{B_i} \\ 1 \end{bmatrix} = \begin{bmatrix} & [T^{i}_{01}] & & \begin{matrix}0\\0\\0\end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_{D_i} - X_A \\ Y_{D_i} - Y_A \\ Z_{D_i} - Z_A \\ 1 \end{bmatrix} + \begin{bmatrix} & [T^{i}_{21}] & & \begin{matrix}0\\0\\0\end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_C \\ y_C \\ z_C \\ 1 \end{bmatrix} \tag{9}$$

$$\begin{bmatrix} \tilde{x}_{C_i} \\ \tilde{y}_{C_i} \\ \tilde{z}_{C_i} \\ 1 \end{bmatrix} = -\begin{bmatrix} & [T^{i}_{02}] & & \begin{matrix}0\\0\\0\end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_A - X_{D_i} \\ Y_A - Y_{D_i} \\ Z_A - Z_{D_i} \\ 1 \end{bmatrix} + \begin{bmatrix} & [T^{i}_{12}] & & \begin{matrix}0\\0\\0\end{matrix} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_B \\ y_B \\ z_B \\ 1 \end{bmatrix}, \tag{10}$$

where $[T^{i}_{kj}]$ is the transfer matrix from the k coordinate system to the j system determined as

$$T^{i}_{01} = [T^{i}_{10}]^T \quad T^{i}_{02} = [T^{i}_{20}]^T \quad T^{i}_{21} = T^{i}_{01} \times T^{i}_{20} \quad T^{i}_{12} = T^{i}_{02} \times T^{i}_{10}, \ldots \tag{11}$$

The necessary conditions for minimum of the sum of squares of the weighted difference

$$S = \sum_{i=1}^{N} \left[\Delta_{q_i}^{(k)}\right]^2 \quad k = 1,2,3 \tag{12}$$

may be written as the following system of equations

$$\frac{\partial S}{\partial X_A} = 0\,,\ \frac{\partial S}{\partial Y_A} = 0\,,\ \frac{\partial S}{\partial Z_A} = 0\,,\ \frac{\partial S}{\partial R} = 0\,, \tag{13}$$

$$\frac{\partial S}{\partial x_B} = 0\,,\ \frac{\partial S}{\partial y_B} = 0\,,\ \frac{\partial S}{\partial z_B} = 0\,,\ \frac{\partial S}{\partial R} = 0\,, \tag{14}$$

$$\frac{\partial S}{\partial x_C} = 0\,,\ \frac{\partial S}{\partial y_C} = 0\,,\ \frac{\partial S}{\partial z_C} = 0\,,\ \frac{\partial S}{\partial R} = 0\,. \tag{15}$$

From (13-15), considering (5-7) and (12), we obtain

$$\left.\begin{array}{c} \sum_{i=1}^{N} \Delta_{q_i}^{(1)} \cdot \left(\tilde{X}_{A_i} - X_A\right) = 0 \\ \sum_{i=1}^{N} \Delta_{q_i}^{(1)} \cdot \left(\tilde{Y}_{A_i} - Y_A\right) = 0 \\ \sum_{i=1}^{N} \Delta_{q_i}^{(1)} \cdot \left(\tilde{Z}_{A_i} - Z_A\right) = 0 \\ \sum_{i=1}^{N} \Delta_{q_i}^{(1)} \cdot R = 0 \end{array}\right\}. \tag{16}$$

Assume that $R \neq 0$. Then from the last equality of the system (16), it follows that

$$\sum_{i=1}^{N} \Delta_{q_i}^{(1)} = 0\,. \tag{17}$$

With provision for (17), the system of equations (16), takes the form:

$$\sum_{i=1}^{N} \Delta_{q_i}^{(1)} \tilde{X}_{Ai} = 0,\ \sum_{i=1}^{N} \Delta_{q_i}^{(1)} \tilde{Y}_{Ai} = 0,\ \sum_{i=1}^{N} \Delta_{q_i}^{(1)} \tilde{Z}_{Ai} = 0,\ \sum_{i=1}^{N} \Delta_{q_i}^{(1)} = 0\,. \tag{18}$$

By substituting the expressions for $\Delta_{q_i}^{(1)}$ from (5) into the system (18), we obtain

$$\left.\begin{array}{c} \sum_{i=1}^{N} \left[\tilde{X}_{Ai}^2 X_A + \tilde{X}_{Ai}\tilde{Y}_{Ai} Y_A + \tilde{Z}_{Ai}\tilde{X}_{Ai} Z_A + \frac{1}{2}\left(R^2 - X_A^2 - Y_A^2 - Z_A^2\right)\tilde{X}_{Ai} \right] = \\ = \frac{1}{2}\sum_{i=1}^{N} \left(\tilde{X}_{Ai}^2 + \tilde{Y}_{Ai}^2 + \tilde{Z}_{Ai}^2\right)\tilde{X}_{Ai} \\ \sum_{i=1}^{N} \left[\tilde{X}_{Ai}\tilde{Y}_{Ai} X_A + \tilde{Y}_{Ai}^2 Y_A + \tilde{Z}_{Ai}\tilde{Y}_{Ai} Z_A + \frac{1}{2}\left(R^2 - X_A^2 - Y_A^2 - Z_A^2\right)\tilde{Y}_{Ai} \right] = \\ = \frac{1}{2}\sum_{i=1}^{N} \left(\tilde{X}_{Ai}^2 + Y_{Ai}^2 + Z_{Ai}^2\right)\tilde{Y}_{Ai} \\ \sum_{i=1}^{N} \left[\tilde{Z}_{Ai}\tilde{X}_{Ai} X_A + \tilde{Y}_{Ai}\tilde{Z}_{Ai} Y_A + \tilde{Z}_{Ai}^2 Z_A + \frac{1}{2}\left(R^2 - X_A^2 - Y_A^2 - Z_A^2\right)\tilde{Z}_{Ai} \right] = \\ = \frac{1}{2}\sum_{i=1}^{N} \left(\tilde{X}_{Ai}^2 + \tilde{Y}_{Ai}^2 + \tilde{Z}_{Ai}^2\right)\tilde{Z}_{Ai} \\ \sum_{i=1}^{N} \left[\tilde{X}_{Ai} X_A + \tilde{Y}_{Ai} Y_A + \tilde{Z}_{Ai} Z_A + \frac{1}{2}\left(R^2 - X_A^2 - Y_A^2 - Z_A^2\right)\tilde{X}_{Ai} \right] = \\ = \frac{1}{2}\sum_{i=1}^{N} \left(\tilde{X}_{Ai}^2 + \tilde{Y}_{Ai}^2 + \tilde{Z}_{Ai}^2\right) \end{array}\right\}. \tag{19}$$

The system (19) is linear with respect to the variables

$$X_A,\ Y_A,\ Z_A \text{ and } H_1 = \frac{1}{2}(R^2 - X_A^2 - Y_A^2 - Z_A^2),$$

thus it may be written as

$$\begin{bmatrix} \sum_{i=1}^{N}\tilde{X}_{Ai}^2 & \sum_{i=1}^{N}\tilde{X}_{Ai}\tilde{Y}_{Ai} & \sum_{i=1}^{N}\tilde{X}_{Ai}\tilde{Z}_{Ai} & \sum_{i=1}^{N}\tilde{X}_{Ai} \\ \sum_{i=1}^{N}\tilde{X}_{Ai}\tilde{Y}_{Ai} & \sum_{i=1}^{N}\tilde{Y}_{Ai} & \sum_{i=1}^{N}\tilde{Y}_{Ai}\tilde{Z}_{Ai} & \sum_{i=1}^{N}\tilde{Y}_{Ai} \\ \sum_{i=1}^{N}\tilde{X}_{Ai}\tilde{Z}_{Ai} & \sum_{i=1}^{N}\tilde{Y}_{Ai}\tilde{Z}_{Ai} & \sum_{i=1}^{N}\tilde{Z}_{Ai}^2 & \sum_{i=1}^{N}\tilde{Z}_{Ai} \\ \sum_{i=1}^{N}\tilde{X}_{Ai} & \sum_{i=1}^{N}\tilde{Y}_{Ai} & \sum_{i=1}^{N}\tilde{Z}_{Ai} & N \end{bmatrix} \cdot \begin{bmatrix} X_A \\ Y_A \\ Z_A \\ H_1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \sum_{i=1}^{N}\tilde{R}_{A_i}^2\tilde{X}_{A_i} \\ \sum_{i=1}^{N}\tilde{R}_{A_i}^2\tilde{Y}_{A_i} \\ \sum_{i=1}^{N}\tilde{R}_{A_i}^2\tilde{Z}_{A_i} \\ \sum_{i=1}^{N}\tilde{R}_{A_i}^2 \end{bmatrix}, \tag{20}$$

where $\tilde{R}_{Ai}^2 = \tilde{X}_{Ai}^2 + \tilde{Y}_{Ai}^2 + \tilde{Z}_{Ai}^2$.

The solution to this system by Cramer's rule is as follows

$$(X_A, Y_A, Z_A, H_1) = \frac{1}{D_1}(D_{X_A}, D_{Y_A}, D_{Z_A}, D_{H_1}) \quad D_1 \neq 0. \tag{21}$$

Similarly, from (13), considering (6) and (17), we obtain a system of linear equations in the unknowns x_B, y_B, z_B, H_2

$$\begin{bmatrix} \sum_{i-1}^{N}\tilde{x}_{Bi}^2 & \sum_{i-1}^{N}\tilde{x}_{Bi}y_{Bi} & \sum_{i-1}^{N}\tilde{x}_{Bi}\tilde{z}_{Bi} & \sum_{i-1}^{N}\tilde{x}_{Bi} \\ \sum_{i-1}^{N}\tilde{x}_{Bi}y_{Bi} & \sum_{i-1}^{N}\tilde{y}_{Bi}^2 & \sum_{i-1}^{N}\tilde{y}_{Bi}\tilde{z}_{Bi} & \sum_{i-1}^{N}\tilde{y}_{Bi} \\ \sum_{i-1}^{N}\tilde{x}_{Bi}\tilde{z}_{Bi} & \sum_{i-1}^{N}\tilde{y}_{Bi}\tilde{z}_{Bi} & \sum_{i-1}^{N}\tilde{z}_{Bi}^2 & \sum_{i-1}^{N}\tilde{z}_{Bi} \\ \sum_{i-1}^{N}\tilde{x}_{Bi} & \sum_{i-1}^{N}\tilde{y}_{Bi} & \sum_{i-1}^{N}\tilde{z}_{Bi} & N \end{bmatrix} \cdot \begin{bmatrix} x_B \\ y_B \\ z_B \\ H_2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \sum_{i=1}^{N}\tilde{R}_{Bi}^2\tilde{x}_{Bi} \\ \sum_{i=1}^{N}\tilde{R}_{Bi}^2\tilde{y}_{Bi} \\ \sum_{i=1}^{N}\tilde{R}_{Bi}^2\tilde{z}_{Bi} \\ \sum_{i=1}^{N}\tilde{R}_{Bi}^2 \end{bmatrix} \tag{22}$$

By solving this system by Cramer's rule, we obtain

$$(x_B, y_B, z_B, H_2) = \frac{1}{D_2}(D_{x_B}, D_{y_B}, D_{z_B}, D_{H_2}) \quad D_2 \neq 0. \tag{23}$$

From (14), considering (7) and (14), we obtain a system of linear equations in the unknowns x_C, y_C, z_C, H_3.

$$\begin{bmatrix} \sum_{i-1}^{N}\tilde{x}_{Ci}^2 & \sum_{i-1}^{N}\tilde{x}_{Ci}\tilde{y}_{Ci} & \sum_{i-1}^{N}\tilde{x}_{Ci}\tilde{z}_{Ci} & \sum_{i-1}^{N}\tilde{x}_{Ci} \\ \sum_{i-1}^{N}\tilde{x}_{Ci}\tilde{y}_{Ci} & \sum_{i-1}^{N}\tilde{y}_{Ci}^2 & \sum_{i-1}^{N}\tilde{y}_{Ci}\tilde{z}_{Ci} & \sum_{i-1}^{N}\tilde{y}_{Ci} \\ \sum_{i-1}^{N}\tilde{x}_{Ci}\tilde{z}_{Ci} & \sum_{i-1}^{N}\tilde{y}_{Ci}\tilde{z}_{Ci} & \sum_{i-1}^{N}\tilde{z}_{Ci}^2 & \sum_{i-1}^{N}\tilde{z}_{Ci} \\ \sum_{i-1}^{N}\tilde{x}_{Ci} & \sum_{i-1}^{N}\tilde{y}_{Ci} & \sum_{i-1}^{N}\tilde{z}_{Ci} & N \end{bmatrix} \cdot \begin{bmatrix} x_C \\ y_C \\ z_C \\ H_3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \sum_{i=1}^{N}\tilde{R}_{Ci}^2\tilde{x}_{Ci} \\ \sum_{i=1}^{N}\tilde{R}_{Ci}^2\tilde{y}_{Ci} \\ \sum_{i=1}^{N}\tilde{R}_{Ci}^2\tilde{z}_{Ci} \\ \sum_{i=1}^{N}\tilde{R}_{Ci}^2 \end{bmatrix}. \tag{24}$$

From which we obtain x_C, y_C, z_C, H_3

$$(x_C, y_C, z_C, H_3) = \frac{1}{D_3}(D_{x_C}, D_{y_C}, D_{z_C}, D_{H_3}) \quad D_3 \neq 0. \tag{25}$$

Eliminating the first four unknowns X_A, Y_A, Z_A, R, based on formula (20), it is possible to bring the system (13-15) to a system of six equations with six unknowns x_B, y_B, z_B, x_C, y_C, z_C, which is convenient to be given as

$$\begin{aligned}
&\sum_{i=1}^{N} \Delta_{q_i}^{(1)} \frac{\partial \Delta_{q_i}^{(2)}}{\partial x_B} = 0 \qquad \sum_{i=1}^{N} \Delta_{q_i}^{(1)} \frac{\partial \Delta_{i}^{(3)}}{\partial x_C} = 0 \\
&\sum_{i=1}^{N} \Delta_{q_i}^{(1)} \frac{\partial \Delta_{q_i}^{(2)}}{\partial y_B} = 0 \qquad \sum_{i=1}^{N} \Delta_{q_i}^{(1)} \frac{\partial \Delta_{i}^{(3)}}{\partial y_C} = 0 \\
&\sum_{i=1}^{N} \Delta_{q_i}^{(1)} \frac{\partial \Delta_{q_i}^{(2)}}{\partial z_B} = 0 \qquad \sum_{i=1}^{N} \Delta_{q_i}^{(1)} \frac{\partial \Delta_{i}^{(3)}}{\partial z_C} = 0
\end{aligned} \tag{26}$$

Apparently, equations of this system are the same as the three equations of the thirteen degree in the three unknown functions given in the work [3], though in this case we have a system of six equations in six unknown functions. Solution of the system (26) is a labor-intensive task, so it is more effective to apply a search algorithm of the minimum of the function S stated below:

1. Give arbitrarily reference points $B^{(0)} \in Q_1$, $C^{(0)} \in Q_2$.
2. Solve the system of linear equations (21) and determine $X_A^{(1)}$, $Y_A^{(1)}$, $Z_A^{(1)}$, $R_1^{(1)}$.
3. Give points $A^{(1)} \in Q$, $C^{(0)} \in Q_2$.
4. Solve the system of equations (23) and determine $x_B^{(1)}$, $y_B^{(1)}$, $z_B^{(1)}$, $R_2^{(1)}$.
5. Give points $A^{(1)} \in Q$, $B^{(1)} \in Q_1$.
6. Solve the system of equations (25) and determine $x_C^{(1)}$, $y_C^{(1)}$, $z_C^{(1)}$, $R_3^{(1)}$.
7. Check $\left|X_A^{i+1} - X_A^{i}\right| \le \varepsilon$ $\left|Y_A^{i+1} - Y_A^{i}\right| \le \varepsilon$ $\left|Z_A^{i+1} - Z_A^{i}\right| \le \varepsilon$ $\left|R^{i+1} - R^{i}\right| \le \varepsilon$
8. If this condition is satisfied, the iterating is completed.
9. If this condition is not satisfied, proceed to item 1 by replacing the reference points $B^{(0)}$ and $C^{(0)}$ for the found points $B^{(1)}$ and $C^{(0)}$.
10. Then check the accuracy of the prescribed function reproduction by analysis of the position IKC *ABCD*.

$$\bar{r}_{D_0} = T_{10} \cdot T_{21} \cdot T_{32} \cdot \bar{r}_{D_3}$$

11. The iterating is completed, if the accuracy of reproduction satisfies the prescribed function.

If it does not satisfy the prescribed accuracy, it is necessary to proceed to item 1 of the given algorithm.

By applying the algorithm, we obtain a decreasing sequence of values of the objective function $S_1^{(1)}, S_2^{(1)}, S_3^{(1)}, S_1^{(2)}, S_2^{(2)}, S_3^{(2)}$ which has a limit equal to the value of the function S at the point of local minimum. When satisfying the inequality

$$\max\left(\left|R^{(i)} - R^{(i-1)}\right|,\ \left|X_A^{(i)} - X_A^{(i-1)}\right|,\ \left|Y_A^{(i)} - Y_A^{(i-1)}\right|,\ \left|Z_A^{(i)} - Z_A^{(i-1)}\right|\right) \le \varepsilon$$

where ε – the prescribed calculation accuracy, the iterating is completed.

Convergence of the suggested algorithm is proved by the *Weierstrass theorem*: for each function $f(x)$, continuous over [*a*, *b*], and any real number $\varepsilon \succ 0$, such a polynomial $p(x)$ may be found that $\left\|P(x) - f(x)\right\| \prec \varepsilon$.

As a result of the problem solution, the points $A(X_A, Y_A, Z_A)$ are determined in the fixed system of coordinates, $B^{(0)} \in Q_1$, $C^{(0)} \in Q_2$, when coinciding the link *BC* with them, we obtain the desired IKC in form of an open four-link chain *ABCD*.

Then we check the accuracy of the prescribed function reproduction by analysis of the position of IKC *ABCD*. If the accuracy of reproduction satisfies the prescribed function, the iterating is completed, and if it does not satisfy the prescribed accuracy, it is necessary to proceed to item 1 of the prescribed algorithm.

When specifying a part of the desired synthesis parameters in various combinations, we obtain different modifications of IKC [9].

- If the coordinates of the point $A(X_{A_i}, Y_{A\,i}, Z_{Ai})$ and Eulerian angles θ_i^1, ψ_i^1, ϕ_i^1 of the solid Q_1 as well as the axes of the point $D_i(X_{D_i}, Y_{D\,i}, Z_{Di})$ and Eulerian angles θ_i^1, ψ_i^1, ϕ_i^1 of the solid Q_2 are specified, we obtain a three-link open chain *ABCD* (Fig. 1). The necessary conditions for the minimum of the sum S in this case take the form

$$\frac{\partial S}{\partial j} = 0 \quad j = x_B, y_B, z_B, R, x_C, y_C, z_C$$

and to find the minimum S we may use the algorithm given above, considering that the parameters X_A, Y_A, Z_A are specified.

If the points $A(X_A, Y_A, Z_A)$ and $D(X_D, Y_D, Z_D)$ are fixed, then as a result of the synthesis of IKC, we obtain a spatial four-link chain *ABCD*.

- Given the coordinates $x_C = y_C = z_C = 0$ of the point $C \in Q_2$, coordinates X_{Di}, Y_{Di}, Z_{Di} of the point D of the solid Q_2 and Eulerian angles θ_i^1, ψ_i^1, ϕ_i^1 of the solid Q_1, and the desired parameters X_A, Y_A, Z_A, R, x_B, y_B, z_B.

The necessary conditions for the minimum of the sum S take the form

$$\frac{\partial S}{\partial j} = 0 \quad j = X_A, Y_A, Z_A, R, x_B, y_B, z_B$$

To find the minimum of the function S we may use again the algorithm given above, considering that $x_C = y_C = z_C = 0$.

- Given coordinates $x_B, y_B, z_B = 0$ of the point *B* of the solid Q_1 and Eulerian angles of the solid Q_2, θ_i^2, ψ_i^2, ϕ_i^2. The original problem reduces to the definition of the sphere of positions of the fixed point C of the solid Q_2 which is the least remote from N (Fig. 1).

The necessary conditions for the minimum of the sum S are

$$\frac{\partial S}{\partial j} = 0 \quad j = X_A, Y_A, Z_A, R, x_C, y_C, z_C .$$

This problem was studied in details in work [10]. For its solution we may also use the algorithm given above, assuming $x_B, y_B, z_B = 0$, but in this special case, the algorithm of the minimum search is absolutely coinciding with the kinematic inversion method.

Thus, as we see, the problem of IKC with spherical kinematic pairs is solved, and their modifications may be used as modules of structural and kinematic synthesis of spatial linkage mechanisms through specified positions of input and output links.

On existence of problem solution of initial kinematic chain synthesis with spherical kinematic pairs

This section of work describes the existence of problem solution of IKC synthesis with spherical kinematic pairs.

The content of problem solution of IKC with spherical pairs is as follows (11)

$$T_{01}^{i}=\left[T_{10}^{i}\right]^{T}\ \ T_{02}^{i}=\left[T_{20}^{i}\right]^{T}\ \ T_{21}^{i}=T_{01}^{i}\times T_{20}^{i}\ \ T_{12}^{i}=T_{02}^{i}\times T_{10}^{i}. \quad (27)$$

By considering the necessary conditions for a minimum sum of the weighted squared differences

$$S=\sum_{i=1}^{N}\left[\Delta_{q_i}\right]^{2}. \quad (28)$$

But, the stated algorithm of synthesis of IKC parameters with spherical kinematic pairs as per specified positions of solids Q_1 and Q_2 does not give results in such cases of degeneracy, when one of the determinants $D_i (i=\overline{1,3})$ in the course of iteration goes to zero (21), (23), (25), therefore there may be cases in the course of IKC synthesis with spherical kinematic pairs, when degeneracy conditions (21), (23) and (25) may be met in various combinations. The total number of variations of indicated combinations, including non-degenerate case as well

$$C_3^0+C_3^1+C_3^2+C_3^3=8.$$

Therefore, during IKC synthesis with spherical kinematic pairs as per specified positions of solids Q_1 and Q_2 we may obtain eight different structural variations of IKC. Thus, at various combinations of $D_i=0\ \ (i=\overline{1,3})$ we have come to the following conclusion.

Theorem: If two adjacent links of open four-link IKC with spherical kinematic pairs go to infinity, it is necessary to change the spherical kinematic pair for the plane or cylindrical pair.

Proof: If in expression (21) $D_1=0$, then $(X_A,Y_A,Z_A)\to\infty$ and the center of the circle approaching the sphere will be lied in the plane or along a straight line. Then, except for the required parameters of IKC $B(x_B,y_B,z_B)\in Q_1$ and $C(x_C,y_C,z_C)\in Q_2$ with a common parameter R, on a fixed system of coordinates *OXYZ* instead of the point $A(X_A,Y_A,Z_A)\in Q$ it is necessary to determine the coefficients a, b, c in the plane q.

$$aX_i+bY_i+cZ_i+1=0\ \ (i=1,N). \quad (29)$$

The synthesized link *AB* limits the movement of the point *B* of solid Q_1 along the plane *q*. Therefore, the coefficients of the point *B* shall satisfy the equation of the plane in specified $N\le 9$ positions. After determining a, b, c coefficients of the plane, it is necessary to select structural parameters of the plane pair. An equivalent kinematic chain with two prismatic pairs is shown in Fig. 2, the axes of which are parallel to the plane *q*.

Similarly, when *N* of the specified positions of solid Q_1 determined by coordinates $(\tilde{X}_{B_i},\tilde{Y}_{B_i},\tilde{Z}_{B_i})\ \ (i=\overline{1,N})$, are lying in the plane (along a straight line), then $D_2=0$.

If $\tilde{X}_{B_i}=\tilde{Y}_{B_i}=\tilde{Z}_{B_i}\neq 0$, i.e. these points do not coincide with the origin of coordinates Axy, the system (21) is also incompatible, x_B, y_B, $z_{B_2}, H_2\to\infty$ and the center of the circle approaching the sphere is infinitely distant. Then in solid Q_1 it is necessary to determine a plane or straight line (23) with coefficients a, b, c, in solid Q_2 the point C with coordinates x_C, y_C, z_C and the point $A(X_A,Y_A,Z_A)$ on a fixed plane. An equivalent four-link kinematic chain ACD with cylindrical and spherical pairs is shown in Fig. 3.

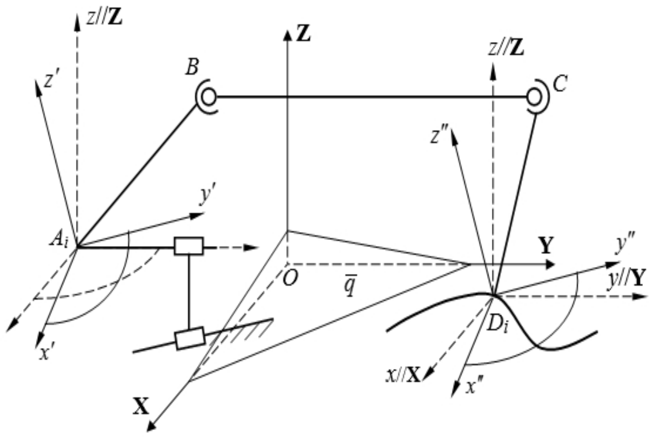

Fig. 2. **Equivalent four-link initial kinematic chain *ABCD***

In the latter case, when all *N* of the specified conditions of synthesis of movable solid Q_2 points determined by coordinates $(\tilde{x}_{C_i}, y_{C_i}, z_{C_i})$, $(i = \overline{1, N})$ are lying along one straight line (in the plane), then $D_3 = 0$..

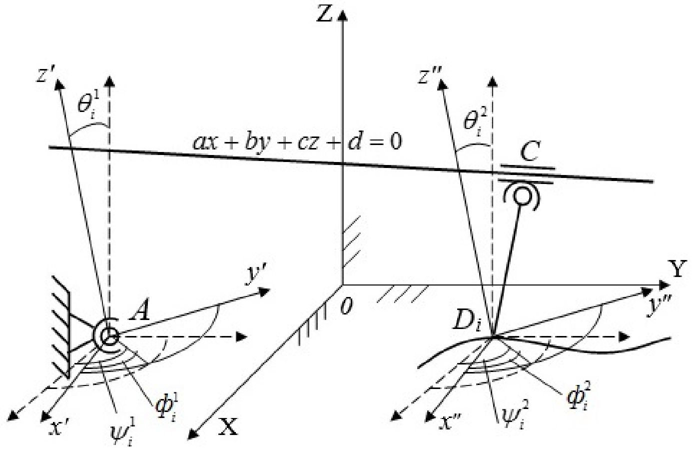

Fig. 3. **Equivalent four-link initial kinematic chain *ACD***

If $\tilde{x}_{C_i} = \tilde{y}_{C_i} = \tilde{z}_{C_i} \neq 0$, i.e. these points do not coincide with the origin of coordinates $Dx'y'z'$, the system (25) is incompatible, $x_C, y_C, z_C, H_3 \to \infty$ and the center of the circle approaching the sphere is infinitely distant. In this case, it is necessary to find a straight line on solid Q_2, on solid Q_1 point $B(x_B, y_B, z_B)$ and on solid *Q* point $A(x_A, y_A, z_A)$ in all specified positions. An equivalent four-link kinematic chain *ABD* with cylindrical and spherical pairs is shown in Fig. 4.

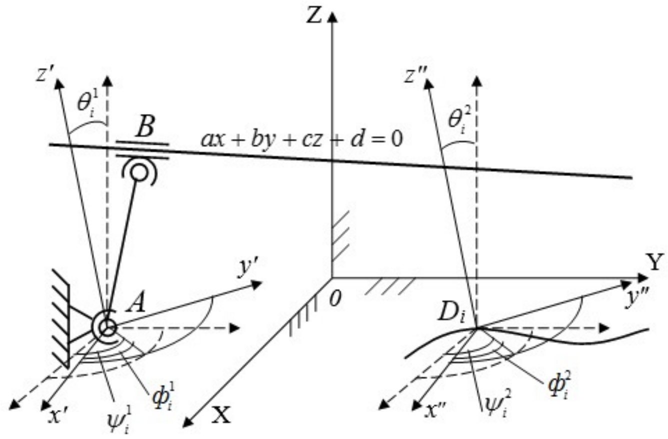

Fig. 4. **Equivalent four-link initial kinematic chain *ABD***

Thus, during synthesis of IKC with spherical kinematic pairs as per specified positions of input and output links of the mechanism, in the case when two adjacent links of IKC go to infinity, it is necessary to replace the spherical kinematic pair for the plane or cylindrical. In such a case, after definition of the required parameters, the synthesized mechanism will have an appearance of a spatial slotted link mechanism.

Synthesis of initial kinematic chain with rotating, plane and spherical kinematic pairs

This section considers a problem of synthesis of spatial initial kinematic chains (IKC) with rotating, plane and spherical kinematic pairs as per specified positions of input and output links based on introduction of two movable solids all the time connected with input and output links [5-8].

It is required to find the point $A(X_A, Y_A, Z_A)$ on solid Q, to find the plane on solid Q_1

$$ax + by + cz + d = 0 \tag{30}$$

and the point $C(x_C, y_C, z_C)$ on solid Q_1, which in its motion about solid Q_1 approached to the desired plane (30). Equation of the plane (30) on fixed solid Q is determined by known transformation formulas

$$a[(X_{Ci} - X_A)t_{11} + (Y_{Ci} - Y_A)t_{21} + (Z_{Ci} - Z_A)t_{31}] + b[(X_{Ci} - X_A)t_{12} + (Y_{Ci} - Y_A)t_{22} + (Z_{Ci} - Z_A)t_{32}] +$$
$$+ c[(X_{Ci} - X_A)t_{13} + (Y_{Ci} - Y_A)t_{23} + (Z_{Ci} - Z_A)t_{33}] + d = 0, \tag{31}$$

where ψ_{ji}, ϕ_{ji} angles are given; angle $\theta_{ji} = 0$, $j = 1,2\ldots$, $i = \overline{1,N}$. $\alpha_{ji} = \psi_{1i} + \phi_{1i}$, $\beta_{ji} = \psi_{2i} + \phi_{2i}$.

$$\begin{bmatrix} X_{Ci} \\ Y_{Ci} \\ Z_{Ci} \\ 1 \end{bmatrix} = \begin{bmatrix} \cos(\psi_{2i} + \phi_{2i}) & -\sin(\psi_{2i} + \phi_{2i}) & 0 & X_{Di} \\ \sin(\psi_{2i} + \phi_{2i}) & \cos(\psi_{2i} + \phi_{2i}) & 0 & Y_{Di} \\ 0 & 0 & 1 & Z_{Di} \\ 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x_C \\ y_C \\ z_C \\ 1 \end{bmatrix}, \tag{32}$$

$$t_{11} = \cos(\psi_{1i} + \phi_{1i}) \;\; t_{21} = \sin(\psi_{1i} + \phi_{1i}) \;\; t_{12} = -\sin(\psi_{1i} + \phi_{1i}) \;\; t_{22} = \cos(\psi_{1i} + \phi_{1i}),$$

$$t_{33} = 1 \;\; t_{13} = t_{31} = t_{23} = t_{32} = t_{41} = t_{42} = t_{43} = 0.$$

After substitution of the expression (32) in formula (31) and required transformations let us comprise the weighted difference Δq_i of the point $C_i(x_C, y_C, z_C)$ from the plane (31) as

$$\Delta q_i = G_1 \cos\alpha_{ji} + G_2 \sin\alpha_{ji} + G_3 \cos(\alpha_{ji} - \beta_{ji}) + G_4 \sin(\alpha_{ji} - \beta_{ji}) + G_5 X_i +$$
$$+ G_6 Y_i + G_7 Z_i + G_8 + G_9 - 1, \tag{33}$$

where

$$G_1 = -(aX_A + bY_A) \;\; G_2 = bX_A - aY_A \;\; G_3 = ax_C + by_C$$

$$G_4 = ay_C - bx_C \;\; G_5 = a \;\; G_6 = b \;\; G_7 = c \;\; G_8 = cz_C \;\; G_9 = -cZ_A$$

$$\begin{bmatrix} X_i \\ Y_i \\ Z_i \end{bmatrix} = \begin{bmatrix} \cos\alpha_{ji} & \sin\alpha_{ji} & 0 \\ -\sin\alpha_{ji} & \cos\alpha_{ji} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} X_{Di} \\ Y_{Di} \\ Z_{Di} \end{bmatrix}.$$

It should be noted that ten required parameters enter into the expression (31), but after normalization of the straight-line equation $d = -1$, the nine required parameters remain. These are the coefficients of equation of the plane a, b, c coordinates X_A, Y_A, Z_A, points $A \in Q$ and coordinates x_C, y_C, z_C, points $C \in Q_2$.

Let us comprise the sum of squares of the weighted difference for N positions

$$S = \sum_{i=1}^{N} [\Delta q_i]^2 \quad i = \overrightarrow{1, N}$$

Stationary conditions per variables

$$\frac{\partial S}{\partial j} = 0 \quad (j = G_1, G_2, G_3, G_4, G_5, G_6, G_7, G_8, G_9)$$

would result in the following simultaneous linear algebraic equations as $G_1 \div G_9$

$$A \cdot \overline{G} = \overline{B} \tag{34}$$

where matrix elements $A(9,9)$, $\overline{G}(9)$, $\overline{B}(9)$.

Solution of the system (34) enables you to determine the required parameters of synthesis. When the coordinates of stand $D(X_D, Y_D, Z_D)$ of the output link of the slotted link mechanism have fixed values, you can determine nine required parameters of synthesis:

$$X_A = -\frac{aG_1 - bG_2}{a^2 + b^2} \quad Y_A = -\frac{bG_1 + aG_2}{a^2 + b^2} \quad Z_A = -\frac{G_9}{G_7} \quad x_C = \frac{aG_3 - bG_4}{a^2 + b^2} \quad y_C = \frac{aG_4 + bG_3}{a^2 + b^2} \quad z_C = \frac{G_8}{G_7}$$

$$a = G_5 \quad b = G_6 \quad c = G_7 \quad a^2 + b^2 \neq 0 \quad c \neq 0$$

Therefore, as per assigned positions of input and output links of transfer mechanisms, you can synthesize the spatial slotted link mechanisms of type RP_LSR (R rotational, P_L Plane, S spherical kinematic pairs).

Now, let us consider a matter of choice of normalization of coefficients of equation of the plane a, b, c With normalization of $d = -1$, we obtain the weighted difference (32). When entering $a = -1$, $b = -1$, $c = -1$ into expression (32), we obtain exact expressions of displacements $(\Delta_i)_x$, $(\Delta_i)_y$, $(\Delta_i)_z$ of points C_i along axes OX, OY, OZ, respectively, which are weighted relative to displacement Δ_i along a normal line. Therefore, you may not say beforehand which normalization is the best and so it would be reasonable to consider all four events.

Example 1

Approximating synthesis of the spatial crank head $RSSS_L$ (S_L Slider) mechanism on the basis of IKC on standard positions of input and output links.

Let us set an angle of rotation ϕ an input link and according to the provision of output link of the projected mechanism (see Fig. 5). We make the weighed difference Δ_{q_i}

$$\Delta_{q_i} = (X_{B_i} - X_{A_i})^2 + (Y_{B_i} - Y_{A_i})^2 + (Z_{B_i} - Z_{A_i})^2 - R^2 \quad i = \overrightarrow{1, N} \tag{35}$$

where $X_{Ai} = a \cos \phi_i$, $X_{Bi} = X_0$,

$Y_{Ai} = a \sin \phi_i$, $Y_{Bi} = S \sin \alpha$,

$Z_{Ai} = 0$, $Z_{Bi} = Z_0 + S \cos \alpha$.

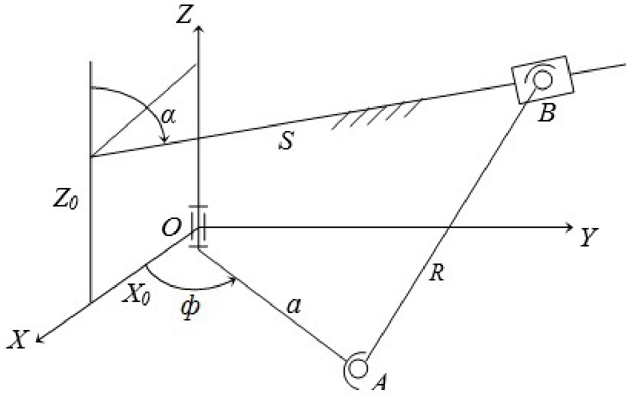

Fig. 5. **Equivalent four-link initial kinematic chain *OAB***

Substituting the values X_A, Y_A, Z_A, x_B, y_B, we find

$$\Delta_{q_i} = (X_0 - a\cos\phi_i)^2 + (S_i a \sin\alpha - a\sin\phi_i)^2 + (Z_0 - S\cos\alpha)^2 - R^2 \quad (36)$$

Simplifying and using trigonometrical identities, we lead expression (36) to a look

$$\Delta_{q_i} = -2aX_0\cos\phi_i - 2a\sin\alpha(S\sin\phi_i) + 2Z_0\cos\alpha\cdot s + X_0^2 + a^2 + Z_0^2 - R^2 + S^2 \quad (37)$$

We enter notation

$$G_1 = 2X_0; G_2 = 2a\sin\alpha; G_3 = 2Z_0\cos\alpha; G_4 = X_0^2 + Z_0^2 + a^2 - R^2$$

the equation (37) takes the form

$$\Delta_{q_i} = -G_1\cos\phi_i + G_2(S_i\sin\phi_i) + G_3 S_i + G_4 + S_i^2 \quad (38)$$

From the necessary condition of minimum of the square sum of the weighed difference

$$S = \sum_{i=1}^{N}[\Delta q_i]^2 \quad i = \overline{1, N} \quad (39)$$

it is possible to write down in the form of the following system of equations

$$\frac{dS}{dj} = 0, \quad (j = G_1 \div G_4) \quad (40)$$

Having written down the equation for $(\phi_i, S_i, i = \overline{1,4})$, we receive a system of four equations with four unknown quantities. After determination of the unknown coefficients of G_1, G_2, G_3, G_4 by the standard decision of the linear system of the equations it is possible to find the demanded design data of the mechanism from the following expression.

$$a = \frac{G_1}{2X_0} \quad \alpha = \arcsin\left(\frac{G_2}{2a}\right) \quad Z_0 = \frac{G_3}{2\cos\alpha} \quad (41)$$

$$R = \left(X_0^2 + Z_0^2 + a^2 - G_4\right)^{1/2}$$

At approximation on six provisions, the angle of rotation of an input link and the position of the output S_1 corresponding to the first exact point is added to four unknown parameters (a, R, X_0, Z_0), defining from the equation (35).

At substitution $S_i = S_0 + S_i$ and $\phi_i = \phi_0 + \phi_i$ in the equation (38) we receive the system from six equations, linear rather unknown G_i, i=$\overline{1,6}$

$$\Delta_{q_i} = -G_1(S_i\cos\phi_i) - G_2(S_i\sin\phi_i) - G_3(\cos\phi_i) + G_4(\sin\phi_i) + G_5 S_i + G_6 + S_i^2 \quad (42)$$

where $G_1 = 2a\sin\alpha\sin\phi_0$, $G_2 = 2a\sin\alpha\cos\phi_0$,

$G_3 = 2aX_0\cos\phi_0 + 2aS_0\sin\alpha\sin\phi_0$,

$G_4 = 2aX_0\sin\phi_0 + 2aS_0$, $G_5 = 2Z_0\cos\alpha + 2S_0$

$G_6 = X_0^2 + Z_0^2 + a^2 - R^2 + S_0^2 + 2Z_0Z_0\cos\alpha$.

Having defined the unknown coefficients of G_1, G_2, G_3, G_4, G_5, G_6, from the necessary condition of minimum of the sum (40) taking into account (41), it is possible to find the values of six required parameters from ratios.

$$\phi_0 = \operatorname{arctg}\frac{G_2}{G_1} \quad a = \frac{G_1}{2}\sin\alpha\sin\phi_0$$

$$X_0 = \frac{\begin{vmatrix} G_3 & 2a\sin\alpha\sin\phi_0 \\ G_4 & -2a\sin\alpha\cos\phi_0 \end{vmatrix}}{\begin{vmatrix} 2a\cos\phi_0 & 2a\sin\alpha\sin\phi_0 \\ 2a\sin\phi_0 & 2a\sin\alpha\cos\phi_0 \end{vmatrix}} \quad S_0 = \frac{\begin{vmatrix} 2a\cos\phi_0 & G_3 \\ 2a\sin\phi_0 & G_4 \end{vmatrix}}{\begin{vmatrix} 2a\cos\phi_0 & 2a\sin\alpha\sin\phi_0 \\ 2a\sin\phi_0 & -2a\sin\alpha\cos\phi_0 \end{vmatrix}}$$

$$Z_0 = \frac{G_5 2S_0}{2\cos\alpha} \quad R = \left(X_0^2 + Z_0^2 + a^2S_0^2 + 2Z_0S_0\cos\alpha\right)^{1/2}$$

Example 2

Let us consider a problem of synthesis of a spatial four-link mechanism of type *RSSR* (*R* – rotational, *S* – spherical kinematic pairs). Approximately reproducing function [9-10].

$$\psi = -50\cos\frac{6}{5}\phi \quad \phi \in \left[0^0, 120^0\right].$$

We divide an interval $\left[0^0, 120^0\right]$ into 20 equal parts (Fig. 6).

If we are given the axial angle $\rho = 90^\circ$ and coordinates $X_D = Z_D = 0$, $Y_D = 1{,}2$ of the point D , let us determine the following parameters of the four-link chain $ABCD$

$$a = \sqrt{x_B^2 + y_B^2 + z_B^2} \quad b = R \quad c = \sqrt{x_C^2 + y_C^2 + z_C^2} \quad \phi_0 = \arccos\frac{z_B}{a} \quad \psi_0 = \arccos\frac{z_C}{c}$$

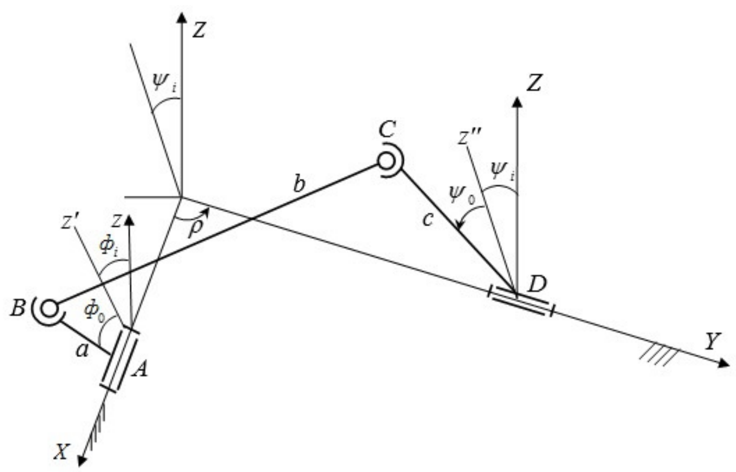

Fig. 6. **Four link spatial chain** *ABCD*

For synthesis of this problem, we use the expression (2). Then the matrix (8-10) is as follows $T_{jk}^i = T_{01} \cdot T_{02} \cdot T_{03}$

where

$$T_{01} = \begin{bmatrix} \cos\phi_i & 0 & -\sin\phi_i \\ 0 & 1 & 0 \\ \sin\phi_i & 0 & \cos\phi_i \end{bmatrix} T_{02} = \begin{bmatrix} \cos\rho & -\sin\rho & 0 \\ \sin\rho & \cos\rho & 0 \\ 0 & 0 & 1 \end{bmatrix} T_{03} = \begin{bmatrix} \cos\psi_i & 0 & \sin\psi_i \\ 0 & 1 & 0 \\ -\sin\psi_i & 0 & \cos\psi_i \end{bmatrix}.$$

In order to determine the required parameters of synthesis, we use the search algorithm of minimum of the sum S. Based on the above minimization algorithm, the value of minimum of the sum S=0,00012. Then a problem of mechanism synthesis reduces to minimization of the objective function of IKC

$$S = \sum_{i=1}^{21} \left[\Delta_{qi} \left(X_A, Y_A, Z_A, x_B, y_B, z_B, R, x_C, y_C, z_C \right) \right]^2 .$$

For solution of this problem we apply the above given search algorithm of minimum of the sum S. According to the algorithm, we are given 10 values $B\left(x_B^{(0)}, y_B^{(0)}, z_B^{(0)}\right)$ и $C\left(x_C^{(0)}, y_C^{(0)}, z_C^{(0)}\right)$ depending on the length of the links a and c. For each preliminary value of the points $B\left(x_B^{(0)}, y_B^{(0)}, z_B^{(0)}\right)$ and $C\left(x_C^{(0)}, y_C^{(0)}, z_C^{(0)}\right)$ in Table 1 the results of calculations are given [11-15].

The iteration process of minimum search of the function S is completed upon satisfaction of equation

$$\left| R^{(k)} - R^{(k-1)} \right| \le \varepsilon \; \left| X_A^{(k)} - X_A^{(k-1)} \right| \le \varepsilon \; \left| Y_A^{(k)} - Y_A^{(k-1)} \right| \le \varepsilon \; \left| Z_A^{(k)} - Z_A^{(k-1)} \right| \le \varepsilon ,$$

where $\varepsilon = 10^{-4}$.

Results and Discussion

Suppose that it is necessary to design a four-linkage mechanism with spherical pairs (Fig. 1), approximately reproducing seven body positions specified in Table 1. Figures 7-10 demonstrate 2D graphics of objective function *S* for Example 2.

Table 1

The specified body positions

№	x_B	y_B	z_B	R	x_C	y_C	z_C	X_A	Y_A	Z_A	S
1	1.014	1.815	1.296	2.640	0.804	1.981	0.327	0.0001	1.204	0	0.0015
2	0.917	1.716	0.821	2.415	0.901	1.761	0.383	0.0002	1.105	0	0.0010
3	-1.117	1.644	0.726	1.967	0.766	0.926	0.414	0	1.086	0.0001	0.0012
4	-0.318	1.471	0.922	1.764	0.546	0.816	0.438	0	1.266	0.0001	0.0005
5	-0.227	1.213	0.609	1.691	0.652	0.639	0.469	0	1.189	0	0.0001
6	0.781	1.044	0.323	1.511	0.591	0.622	0.455	0	1.099	0.0001	0.0001
7	0.191	1.166	0.221	1.724	0.492	0.604	0.718	0.0003	1.675	0	0.0002
8	0.212	1.071	0.348	1.547	0.515	0.765	0.665	0.0002	1.761	0	0.0014
9	0.561	1.008	0.941	1.334	0.342	0.640	0.684	0.0001	1.360	0.0002	0.0019
10	0.867	1.261	0.646	1.266	0.561	0.762	0.697	0.0001	1.141	0.000	0.0003

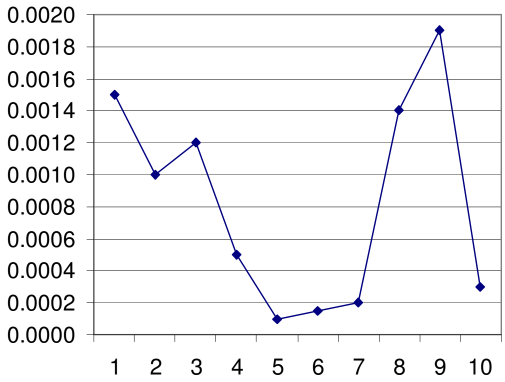

Fig. 7. **Number of points *S***

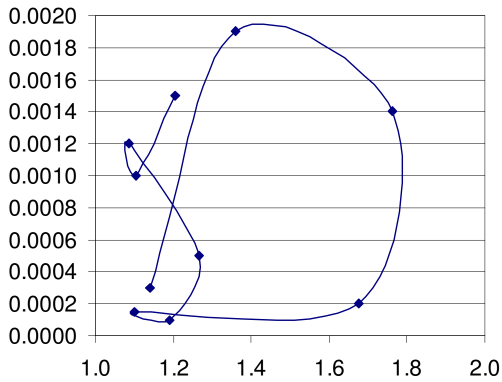

Fig. 8. **Number of function points $S = S(Y_A)$**

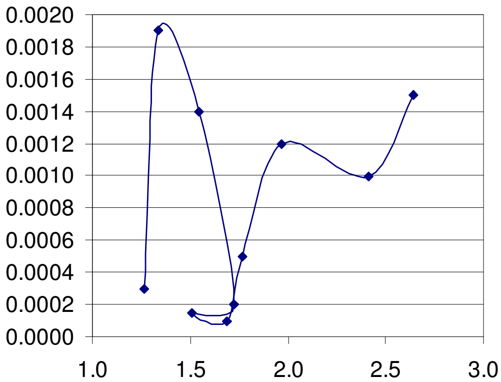

Fig. 9. **Number of function points $S = S(R)$**

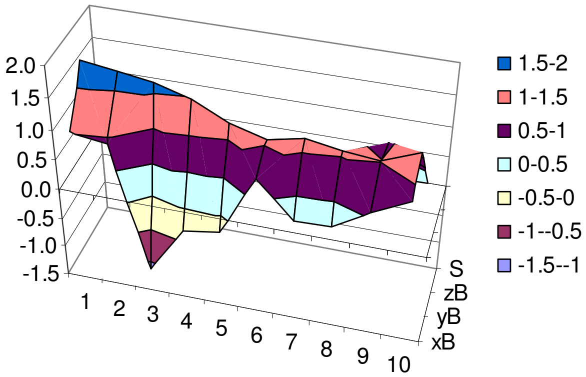

Fig. 10. **Graphics of function $S = S(x_B, y_B, z_B)$**

According to the algorithm the value of the global minimum $S_{\min} = 0.004292$ has been determined as well as the value of coordinates of the global minimum:

$$x_B = -0.227631 \quad x_C = 0.652906 \quad X_A = Z_A = 0$$

$$y_B = 1.21314 \quad y_C = 0.639147 \quad Y_A = 1.189776$$

$$z_B = 0.609801 \quad z_C = 0.469694 \quad R = 1.69127$$

Conclusions

1. The use of one and the same objective function, composed for the synthesis of the IKC and its modification, allows you to automate the process of synthesis of spatial linkage mechanisms as per specified positions of the input and output links of the mechanism.
2. In summary, when there is synthesis of IKC with spherical kinematic pairs as per predetermined positions of the input and output links of the mechanism, and when two adjacent links of IKC are tending to infinity, it is necessary to replace the spherical kinematic pair for a plain or cylindrical. In this case, the synthesized mechanism takes a form of a spatial link mechanism after determining the required parameters.

References

1. Joldasbekov W.A., Drakunov Y.M., Kosbolov S.B., Moldabekov M.M. Synthesis of starting kinematics mechanisms. high grade Math. Academy of Sciences of the Kazakh SSR, series Sci, № 3, 1978, pp. 65-70.
2. Kosbolov S. B., Tanzharikova G. P., Zhauyt A., Rakhmatulina A. B. Kinematical synthesis of six-link linkage with three dwells. Vestnik KazNTU, Almaty, Kazakhstan, №6 (94), 2012, pp. 44-52.
3. Kosbolov S. B., Rakhmatulina A. B. Parametric synthesis of spatial linkage based on the initial kinematic chain. Modern problems of science and education, №2, 2012, pp. 25-36.

4. Sargsyan Y.L. Approximation synthesis mechanisms: Science, 1982, pp. 303-309.
5. Kosbolov S., Moldabekov M., Bekenov E. Kinematic synthesis of spatial lever motion – generating mechanisms by use of initial kinematic chain. The NINTH IFToMM international symposium on teori of machines and mechanisms. Romania, September 1-4, SYROM 2005, pp. 245-250.
6. Vorobyov E.I., Egorov O.D., Popov S.A. Mechanics of industrial robots. Calculation and design of mechanisms. High School, Vol. 2. 366, 1988, pp. 120-127.
7. McCarthy J.M. The synthesis of planar RR and spatial CC chains and the equation of a triangle. Trans. ASME. J. Mech. Des, Suppl. "50th anniv. des. eng. div." 1995, pp. 101-106.
8. Golynski Z. Optimal synthesis problems solved by means of nonlinear programming and random methods. Journal of mechanisms. Vol. 5. №3, 1970, pp. 287-309.
9. Innocenti C. Direct kinematics in analytical form of the 6-4 fully – parallel mechanisms. Trans. ASME. J. Mech. Des, №1 (117), 1995, pp. 85-95.
10. Kosbolov S.B., Rakhmatulina A.B. Kinematic synthesis of three-dimensional six-link motion-generating mechanisms in the basis of initial kinematic chains. Journal of machinery manufacture and reliability. №2, 2013, pp. 102-108.
11. Kosbolov S.B., Rakhmatulina A.B. Synthesis mechanism to compensate weft threads on a multicolored weaving loom machine shuttleless. "Tekstil ve Konfeksiyon", Turkey, vol. 23, 2013, pp. 23-31.
12. Kosbolov S.B., Bekenov E.T. Kinematic synthesis of spatial six-membered transfer mechanism based on the original kinematic chains. Bulletin of Science and Technological Development of Russia Academy of Sciences, Moscow, №6 (70), 2013, pp. 30-38.
13. Kosbolov S.B., Rakhmatulina A.B. Optimization flow of force of plane leverage mechanisms. International conference on European science and technology. Wiesbaden, Germany, 2012, pp. 271-276.
14. Kosbolov S.B., Rakhmatulina A.B. Design of a new load lifting mechanism. Prosedia sogial and behavioral sciences. Vol. 83, 2013, pp. 689-692.
15. Zhauyt A., Kosbolov S., Shingissov B., Alymbetov A., Telesheva A., Karabashev O., Tashkenbayev A. Synthesis of four-link basic kinematic chains [BKC] with spherical pairs for spatial mechanisms. Mediterranean journal of social sciences, Rome-Italy, Vol. 5, №23, 2014, pp. 2627-2637.