NOVEL INSULATION MATERIAL

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Abstract. In circumstances, when it is important to replace insulation materials with high content of emissions during production, it is necessary to create a new heat and sound insulation material, which eliminates CO_2 emissions, develop its production techniques and technological machinery – raw material chopper, pulp mixer, termopress, dryer chamber, formatting knives, determine technical control parameters and control equipment, develop a mathematical model of the material and calculation methods for design works. It is necessary to design, manufacture and experimentally test the respective technological equipment for insulation production pilot plant. To get exact physical parameters it is necessary to design, manufacture and test unique laboratory equipment for determining the properties of the insulation material.

Keywords: insulation material, mathematical model, properties of insulation material.

Introduction

New insulation has been developed by the Liepaja University scientists. It is based on mix of insulation material particles enclosed in fibrous mass having insulation properties (as it contains trapped air micro pockets) and in the same time it bonds the insulation particles, forming a self-supporting layer of the insulation material useful both for thermal and for acoustic insulation. A remarkable positive property among the others is the ability of the material to accept and release water vapour – "breathe" like most of the natural materials. Another – it is stable against setting – opposite to pure cellulose wool insulation.

Materials and mathematical model

Appearance of new insulation material is shown on Fig 1. And Fig. 2.Insulation foam particles are incorporated in between fibrous material layers and aggregated with no adhesive additives.

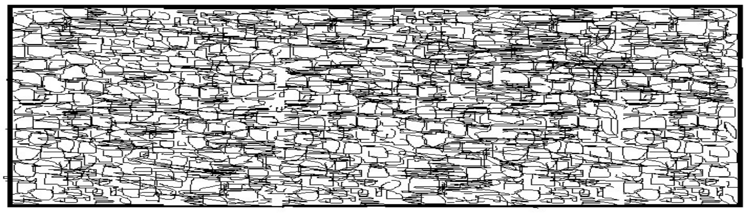

Fig. 1. **Structure of insulation material: consists of "foam" insulation particles and uneven fiber layers**

Fig. 2. **Foamed polystyrene particles are bonded with cellulose fibers**

In this section we propose the mathematical model describing the dynamics of propagation and retention of heat over fibre insulation coating by taking "inner" specificities (graininess and porosity of layered structure of the considered fibre insulation; see, for instance, [1-3] as well as [4]) of the heat insulator into account. It should be noted that the proposed model has its limitations: it describes only "internal" physical processes and includes:

- heat propagation in the insulation material;
- mechanical process related to tensions in the material structure and differences in elasticity of the said material under the influence of uneven heat spreading in the insulation material, which has been regarded as non-homogeneous layered structure.

Thus, the proposed mathematical model has the following statement:

1. Four equations concerning the sought-for functions $T = T(x, y, z; t)$ and $u(x, y)$

$$\rho \cdot c \cdot \frac{\partial T}{\partial t} + \nabla_{x,y,z}\left\{k \cdot \nabla_{x,y,z} T\right\} = f,$$
$$(x, y, z; t) \in \operatorname{int} D \times (0, t_{end}]; \quad (1)$$

$$2 \cdot \frac{\partial^2 u_{xx}}{\partial x^2} + \frac{\partial^2 u_{xx}}{\partial y^2} + \frac{\partial^2 u_{yy}}{\partial x^2} E_l \cdot \alpha_l \cdot \left\{ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right.$$
$$\left. - \frac{\partial^2 T_0}{\partial x^2} - \frac{\partial^2 T_0}{\partial y^2} \right\} = 0, \ (x, y, z; t) \in \operatorname{int} D \times (0, t_{end}]; \quad (2)$$

$$\frac{\partial u_{xx}}{\partial x} + \frac{\partial u_{xy}}{\partial y} = 0, \ (x, y, z) \in \operatorname{int} D; \quad (3)$$

$$\frac{\partial u_{xy}}{\partial x} + \frac{\partial u_{yy}}{\partial y} = 0, \ (x, y, z) \in \operatorname{int} D; \quad (4)$$

2. Initial condition

$$T\big|_{t=0+0} = T_0(x, y, z), \ (x, y, z) \in D; \quad (5)$$

3. Eighteen boundary conditions concerning both the thermal field $T = T(x, y, z; t)$ (six boundary conditions) and the thermoelasticity $u(x, y)$ (twelve boundary conditions)

$$T\big|_{x=0+0} = T_{0x}(y, z; t),$$
$$(y, z; t) \in \left(D/[0, L_x]\right) \times [0, t_{end}]; \quad (6)$$

$$T\big|_{x=L_x-0} = T_{Lx}(y, z; t),$$
$$(y, z; t) \in \left(D/[0, L_x]\right) \times [0, t_{end}]; \quad (7)$$

$$T\big|_{y=0+0} = T_{0y}(x, z; t),$$
$$(x, z; t) \in \left(D/[0, L_y]\right) \times [0, t_{end}]; \quad (8)$$

$$T\big|_{y=L_y-0} = T_{Ly}(x, z; t),$$
$$(x, z; t) \in \left(D/[0, L_y]\right) \times [0, t_{end}]; \quad (9)$$

$$T\big|_{z=0+0} = T_{0z}(x, y; t),$$
$$(x, z; t) \in \left(D/[0, L_z]\right) \times [0, t_{end}]; \quad (10)$$

$$\left.\frac{\partial T}{\partial z}\right|_{z=L_z-0} = T_{Lz}(x, y; t),$$
$$(x, z; t) \in \left(D/[0, L_z]\right) \times [0, t_{end}]; \quad (11)$$

$$u_{xx}\big|_{x=0+0} = u_{xx}^{0x}(y),\ \ y \in \left[0, L_y\right]; \tag{12}$$

$$u_{xx}\big|_{x=L_x-0} = u_{xx}^{Lx}(y),\ \ y \in \left[0, L_y\right]; \tag{13}$$

$$u_{xx}\big|_{y=0+0} = u_{xx}^{0y}(x),\ \ x \in \left[0, L_x\right]; \tag{14}$$

$$u_{xx}\big|_{y=L_y-0} = u_{xx}^{Ly}(x),\ \ x \in \left[0, L_x\right]; \tag{15}$$

$$u_{xy}\big|_{x=0+0} = u_{xy}^{0x}(y),\ \ y \in \left[0, L_y\right]; \tag{16}$$

$$u_{xy}\big|_{x=L_x-0} = u_{xy}^{Lx}(y),\ \ y \in \left[0, L_y\right]; \tag{17}$$

$$u_{xy}\big|_{y=0+0} = u_{xy}^{0y}(x),\ \ x \in \left[0, L_x\right]; \tag{18}$$

$$u_{xy}\big|_{y=L_y-0} = u_{xy}^{Ly}(x),\ \ x \in \left[0, L_x\right]; \tag{19}$$

$$u_{yy}\big|_{x=0+0} = u_{yy}^{0x}(y),\ \ y \in \left[0, L_y\right]; \tag{20}$$

$$u_{yy}\big|_{x=L_x-0} = u_{yy}^{Lx}(y),\ \ y \in \left[0, L_y\right]; \tag{21}$$

$$u_{yy}\big|_{y=0+0} = u_{yy}^{0y}(x),\ \ x \in \left[0, L_x\right]; \tag{22}$$

$$u_{yy}\big|_{y=L_y-0} = u_{yy}^{Ly}(x),\ \ x \in \left[0, L_x\right]. \tag{23}$$

In the proposed model (1)-(23) there are the following notations and assumptions:

- $D \overset{def}{\equiv} \left\{(x, y, z) : x \in \left[0, L_x\right], y \in \left[0, L_y\right], z \in \left[0, L_z\right]\right\}$ is the geometric configuration (the closed 3D domain) of the insulation material that has a rectangular shape with the thickness/depth L_z;
- L_x and L_y are the length and the width of the rectangular insulation material, respectively;
- $\operatorname{int} D$ is an open domain that signifies interior of the domain D: $\operatorname{int} D \overset{def}{\equiv} D/\partial D$, ∂D contains the frontier points of the D;
- t_{end} is the time within a period of which we investigate the thermal processes occurring interior of the insulation material;
- the sought-for function $T = T(x, y, z; t)$ is the temperature (or rather the thermal field) in the considering point (x, y, z) of the insulation material at the time moment t;
- the prescribed function $k = k(x, y, z) > 0$ is the heat conductivity coefficient in the considering point (x, y, z) of the insulation material;
- the prescribed function $\rho = \rho(x, y, z) > 0$ is the density of the insulation material in the considering point (x, y, z) of the insulation material;
- the prescribed function $c = c(x, y, z) > 0$ is the specific thermal capacity of the insulation material in the considering point (x, y, z) of the insulation material;
- the prescribed function $f = f(x, y, z; t)$ is the power density of external heat sources applied to the considering point (x, y, z) of the heat insulator at the time moment t;

- the sought-for functions $u_{xx} = u_{xx}(x, y)$, $u_{xy} = u_{xy}(x, y)$ and $u_{yy} = u_{yy}(x, y)$ are the components of mechanical stress and thermoelasticity $u(x, y)$ under assumption of ignoring any changes / influences in the direction of the axis OZ;
- $\nabla_{x,y,z} T \overset{def}{\equiv} \frac{\partial T}{\partial x} \cdot \vec{e}_1 + \frac{\partial T}{\partial y} \cdot \vec{e}_2 + \frac{\partial T}{\partial z} \cdot \vec{e}_3$ is the gradient of the thermal-vector field, where $\vec{e}_i$ $\left(i = \overline{1,3}\right)$ are unit vectors located on the coordinate axes (OX, OY, OZ), respectively;
- the prescribed function $\alpha_l = \alpha_l(x, y, z) \overset{def}{\equiv} \frac{\Delta l}{l \cdot \Delta T}$ is the linear thermal expansion coefficient (see, for instance, [2]);
- the prescribed function $E_l = E_l(x, y, z) \overset{def}{\equiv} \frac{F \cdot l}{S \cdot \Delta l}$ is the modulus of elongation (so-called "Young modulus"; see, for instance, [1; 5]), which characterizes the deformation taking place in the considering point (x, y, z) of the heat insulator surface S under the impact of temperature voltage both sides relative to the heat insulator (i.e. from the outside and on the inside of premises): this Young modulus characterizes also the properties of the heat insulator to make resistance to tension at the elastic deformation under the impact of the temperature voltage;
- the prescribed function $T_0 = T(x, y, z; t)\big|_{t=0+0}$ is the initial temperature in the considering point (x, y, z) of the insulation material;
- the boundary functions $T_{0x}(y, z; t)$, $T_{Lx}(y, z; t)$, $T_{0y}(x, z; t)$, $T_{Ly}(x, z; t)$, $T_{0z}(x, y; t)$, $T_{Lz}(x, y; t)$, $u_{xx}^{0x}(y)$, $u_{xx}^{Lx}(y)$, $u_{xy}^{0y}(x)$, $u_{xy}^{Ly}(x)$, $u_{yy}^{0y}(x)$, $u_{yy}^{Ly}(x)$, $u_{xy}^{0x}(y)$, $u_{xy}^{Lx}(y)$, $u_{yy}^{0x}(y)$ and $u_{yy}^{Lx}(y)$ are assumed as known functions in their applicable / definitional domains.

In order to make sure that the system of four equations (1)-(4), which describes the interrelated processes generating temperature field $T(x, y, z; t)$ and thermoelasticity $u(x, y)$, had a physical determinacy (i.e. physical meaning), it is necessary to have some initial information of quantitative and qualitative patterns (see, for instance, [1; 6; 7-9] and respective references given in these). The initial condition (5) and the boundary conditions (6)-(23) form the required quantitative information. As regards the required information of the qualitative pattern, its forming mostly depends on the chosen methods of analysis and solving the constructed model. Obviously, the less constrains are imposed on the model, the wider the range of application of this model becomes. Therefore, it makes sense to pose a question on finding the optimal set of constrains of the qualitative pattern. However, in view of the fact that we cannot solve the proposed model (1)-(23) in the present paper, the formulated below constrains of the qualitative pattern are conditioned only by mathematical correctness of the equations (1)-(4):

- $T(x, y, z; t) \in C^{1,2}\left\{\text{int}\, D \bigcup [0, t_{end}]\right\}$;
- $u(x, y) \in C^2\left\{\text{int}\, D / \ (0, L_Z)\right\}$;
- $T_0(x, y, z) \in C^2\{D\}$.

Thus, the proposed model (1)-(23) is the complete statement of the initial-boundary-value problem for investigation of the dynamics of propagation and retention of heat over fibre insulation coating by taking "inner" specificities of the heat insulator into account. The analytical and/or numerical solution of the proposed model (1)-(23) will allow finding the sought-for functions $T(x, y, z; t)$ in the domain D and $u(x, y) \overset{def}{\equiv} \left(u_{xx}(x, y), u_{xy}(x, y), u_{yy}(x, y)\right)$ in the domain $D / [0, L_z]$, and consequently, in the time-interval $[0, t_{end}]$, during which the thermal processes occurring interior of the insulation material are investigated, we can completely determine the thermal field and the thermoelasticity of the considered insulation material having a "parallelepiped" shape with spatial measurements $L_x \times L_y \times L_z$.

Conclusions

In this paper we formulated boundary conditions, among which only one is the second kind boundary condition (the so-called Neumann condition), while others are the Dirichlet boundary conditions. This is due to the practical point of view: as a rule an experimental method, which allows realizing boundary functions of the first kind, is an easier technique (however, such approach is not always expedient!). In addition, in the present paper we formulated three conditions / constraints of qualitative pattern (one of many possible variants for constructing the qualitative constraints), the implementation of which together with the formulated boundary conditions and the initial condition unambiguously ensures mathematical correctness of the proposed model.

Qualitative analysis of the proposed model and/or its solving can favour for profound investigation and understanding of the thermal and thermomechanical processes occurring in the insulation materials, and thereby can improve the functionality and reliability of heat insulators.

To conclude with, let us note that the author of this paper is intended to continue further investigation taking the benefit of both qualitative and quantitative studies for the proposed model (1)-(23) as well as to develop stable analytical and numerical methods for their solution ensuring the corresponding computer-based implementation.

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