THEORETICAL STUDY OF MATERIAL DRYING COEFFICIENT

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Abstract. The purpose of this work is theoretically based methodology to determine the drying coefficient of porous material. Using the mathematical model of thick grain layer drying process, a methodology of porous material thin layer drying coefficient determination, using the experimental data, is done. The necessary drying time with constant conditions and linear drying rate is presented. Based on the experimental data, the time dependency of the moisture content and drying rate were calculated and presented. Using the drying coefficient, the layer of apple slice drying was simulated and the obtained results compared with the experimental data. The obtained measurement results are in high correlation with the calculations. The resulting drying rate can be used for modeling the drying process in a layer containing small particles, it is to numerically solve the system of partial differential equations including the matter and environmental temperatures ($\Theta(x, t)$ and $T(x, t)$) and mass exchange ($W(x, t)$, $d(x, t)$).

Keywords: drying coefficient, mathematical modeling.

Introduction

Drying research is an outstanding example of a very complex field where it is necessary to look comprehensively on the simultaneous energy and mass transfer process that takes place within and on the surface of the material. In order to get the full view of the drying process, researches have to incorporate and deal with highly non linear physical phenomena inside drying agricultural products, non- homogenous distribution of temperature and humidity inside the dryers, equipment selection, product final quality. That is the reason why a unique theoretical setting of drying has to be determined through the balance of heat flow, temperature changes and moisture flow.

In order to find optimal drying regime it is necessary to understand the transport mechanisms which take place within and on the surface of the product. The drying process is characterized by the existence of transport mechanisms such as surface diffusion, pure diffusion, capillary flow, evaporation, thermo-diffusion, etc.

There are so many transport mechanisms, that we must choose which of the mechanisms are essential and which are not. It is also important to know what kinds of products under what conditions are dried. Moisture transfer within the solid body at a certain temperature is realized due to the different moisture content in the interior and on the surface of solid body. The mass transfer rate is proportional to the concentration gradient of the moisture content, with the diffusion coefficient being the proportionality factor. Determination of the diffusion coefficient is essential for credible description of the mass transfer process, described by the Fick's equation [1]. It is a common practice to describe complete mass transfer with the same equations as pure diffusion and take the correction for all secondary types of mass transfer into account simply replacing the pure diffusion coefficient with an effective diffusion coefficient.

The situation is different if we look at the porous layer dried with forced ventilation, found in grain drying. With thick layers of materials we need to take into account both the material temperature and humidity variables and agent-ventilated temperature and moisture changing nature [2].

Many studies were done to process agricultural product drying. Several researches have been carried out in the influence of some process parameters (temperature, sample thickness, air flow rate, etc). The effect of carrot slices on the drying kinetics was studied in [3; 4]. The modeling of carrot cubes was made in [5; 6] where the author studied the influence of air-flow rates and the effect of temperature to the drying curve for carrot cubes. The cocoa bean drying process was investigated at [7], where different semi-theoretical models are derived and tested. Influence of pretreatment on the drying rate of chili pepper at various air temperatures was investigated in [8].

The aim of this research was to theoretically find the porous product drying rate using the mathematical model of porous production layer drying with active ventilation. This paper presents methodology used to find the theoretical drying rate and the necessary drying time with the linear drying coefficient.

Materials and methods

Drying of any substance is based on heat-mass transfer processes. In our situation it is based on heat-mass transfer between porous material and inter-material space (air in our situation). Since the porous material thickness is very small, the internal diffusion in the drying process can be ignored.

We propose a mathematical model which contains temperature and moisture functions of the matter and inter-matter space (air). To describe the kinetics of the drying process we assume the following:

- water evaporation in slices of carrot proceeds according to the Dalton law,
- water is liquid in material,
- heat transfer between the matter and the drying agent (air) goes on by convection,
- air flow takes place due to convection and its velocity is constant in the layer of matter,
- inner temperature gradient for single matter pieces is very small and has not been considered.

The heat-mass transfer model is based on laws of physics, i.e., the mass transfer law between the matter and the drying agent, the law of substance conservation, the law of heat transfer between the matter and air and the law of energy conservation. We obtained the following system of partial differential equations including the matter temperature $\Theta(x, t)$, moisture $W(x, t)$ and environmental (air) temperature $T(x, t)$ and humidity $d(x, t)$ [2]:

$$\frac{\partial W}{\partial t} = K\left(W_p - W\right), \quad t > 0, \quad x > 0 \tag{1}$$

$$\frac{\partial d}{\partial t} + a_1 \frac{\partial d}{\partial x} = \frac{k}{a_2}\left(W - W_p\right), \quad t > 0, \quad x > 0 \tag{2}$$

$$\frac{\partial \Theta}{\partial t} = c_1\left(T - \Theta\right) + c_2\left(W_p - W\right), \quad t > 0, \quad x > 0 \tag{3}$$

$$\frac{\partial T}{\partial t} + a_1 \frac{\partial T}{\partial x} = c_0\left(\Theta - T\right) \quad t > 0, \quad x > 0 \tag{4}$$

where x, t – variable of space and time.

There

$$a_1 = 3600\nu, \; a_2 = \frac{\gamma_a \varepsilon}{10\gamma_m}, \; c_0 = \frac{\alpha_q}{m\gamma_a c_a}, \; c_1 = \frac{\alpha_q}{(m-1)\gamma_m c_m},$$

$$c_2 = \frac{Kr}{100c_g}, \; \alpha_q = 12.6\frac{\lambda}{L^2}, \; K = f(T, x).$$

Notations are:

- ν – air velocity , $m \cdot s^{-1}$;
- γ_a, γ_m – capacity of weight /air, matter respectively/, $kg \cdot m^{-3}$;
- c_a, c_m – heat of drying air and moist matter, $kJ \cdot kg^{-1}$;
- r – latent heat for water evaporation, $kJ \cdot kg^{-1}$;
- $\varepsilon = m/(1 - m)$ (m – porosity of matter);
- W_p – equilibrium moisture content, dry basis, %;
- K – drying coefficient, h^{-1};
- α_q – interphase heat exchange coefficient, $kJ \cdot m^{-2} \cdot h^{-1} \cdot {}^{\circ}C^{-1}$;
- λ – rate of matter heat transfer, $kJ \cdot m^{-2} \cdot h^{-1} \cdot {}^{\circ}C^{-1}$;
- $2L$ – material pieces thickness, m.

Equilibrium moisture content W_p was obtained using S. Henderson's modified equation in Forte's interpretation :

$$W_p = \left(-\frac{1}{5869} \ln\left(1 - \frac{\varphi}{100}\right)(T+273)^{0.775} \right)^{\frac{(T+273)^{1.363}}{5203}} ,$$

where φ – heated air relative humidity, %.

Initial and boundary conditions for the system (1) - (4) can be given in the following way:

$$T\big|_{t=0} = \Theta\big|_{t=0} = \Psi_1(x) \ , \ W\big|_{t=0} = \Psi_2(x) \ , \ d\big|_{t=0} = \Psi_3(x)$$

$$T\big|_{x=0} = \vartheta_1(t) \ , \ d\big|_{x=0} = \vartheta_2(t) \ . \tag{5}$$

Initial conditions for the system (1)-(4) at our situation are given as follows:

$$\Psi_1(x) = \Theta_s \ , \ \Psi_2(x) = W_s \, , \ \Psi_3(x) = d_s$$

where Θ_s – matter and inter-matter air temperature in the layer, ºC;
W_s, d_s – matter moisture and inter-matter air humidity in the layer, %.

For material drying we chose constant boundary conditions:

$$\vartheta_1(t) = T_r \, , \ \vartheta_2(t) = d_r$$

where T_r – heated air temperature, ºC;
d_r – heated air humidity, %.

The system (1)-(4) with boundary and initial conditions (5) can be solved numerically by difference scheme using weights [2].

Results and discussion

One of the most important tasks is to find the expression for the drying coefficient K. It depends on both, the dying product, drying equipment, conditions, etc. We assume (for thin product layer and constant boundary condition) W depending only on the drying time t. Its means $T = \Theta$ and variables do not depend on the layer thickness x. The equation (1) becomes

$$\frac{dW}{dt} = K(t) \cdot \left(W_p - W\right), \quad \text{with condition } W(0) = Ws \tag{6}$$

Writing problem (6) as

$$\frac{dW}{W_p - W} = K(t) \cdot dt$$

and solving it, we receive solution

$$\ln\left|W - W_p\right| = -\int K(t)dt + C \ .$$

The condition expressed $C = A + \ln\left|W_s - W_p\right|$, where A is $-\int K(t)dt$ value at t=0. Putting this expression in solution, we find

$$\ln\left|\frac{W - W_p}{W_s - W_p}\right| = A - \int K(t)dt$$

and finally we have

$$W = (W_s - W_p) \cdot \exp[A - \int K(t)dt] + W_p \tag{7}$$

If to the choose drying coefficient as polynomial, it is,

$$K(t) = a_n t^n + a_{n-1} t^{n-1} + ... + a_1 t + a_0$$

we can integrate $K(t)$

$$\int K(t)dt = \frac{a_n}{n+1} t^{n+1} + \frac{a_{n-1}}{n} t^n + ... + \frac{a_1}{2} t^2 + a_0 t$$

and express A = 0. The solution (7) is

$$W = (W_s - W_p) \cdot \exp[-(\frac{a_n}{n+1} t^{n+1} + \frac{a_{n-1}}{n} t^n + ... + \frac{a_1}{2} t^2 + a_0 t)] + W_p \tag{8}$$

How to find $a_n, a_{n-1}, ..., a_1, a_0$?

If we take $u = W_p - W$ and put it in equation (6) we receive $\frac{du}{dt} = -K(t) \cdot u$. Solve this equation and express it as

$$-\frac{\ln u}{t} = \frac{a_n}{n+1} t^n + \frac{a_{n-1}}{n} t^{n-1} + ... + \frac{a_1}{2} t \quad + a_0 . \tag{9}$$

We can receive the values of the left side of (9) from experimental data ($W = W_{\text{exp}}$) for each drying time moment t_{exp} and we can determine the right side expression by the least square method using the experimental values from the left side of equation (9). Using the obtained right side expression we can calculate $a_n, a_{n-1}, ..., a_1, a_0$.

Take a linear dependence of the drying rate $K(t) = a_1 t + a_0$. The product moisture changes can be determined:

$$W = (W_s - W_p) \cdot \exp[-(\frac{a_1}{2} t^2 + a_0 t)] + W_p$$

The important question is "How long must you dry the product if its moisture becomes W_b?" We must calculate the drying time when $W = W_b$ ($W_b > W_p$). Using (8) for our situation we receive

$$a_1 t^2 + 2a_0 t + 2\ln\left|\frac{W_b - W_p}{W_s - W_p}\right| = 0 \quad .$$

Solve the quadratic for t:

$$t_{1,2} = \frac{-2a_0 \pm \sqrt{4a_0^2 - 8a_1 \ln\left|\frac{W_b - W_p}{W_s - W_p}\right|}}{2a_1} = -\frac{a_0}{a_1} \pm \sqrt{\left(\frac{a_0}{a_1}\right)^2 - \frac{2}{a_1}\ln\left|\frac{W_b - W_p}{W_s - W_p}\right|} .$$

As t>0, the necessary drying time with constant conditions and linear drying rate is

$$t = \sqrt{\left(\frac{a_0}{a_1}\right)^2 - \frac{2}{a_1}\ln\left|\frac{W_b - W_p}{W_s - W_p}\right|} - \frac{a_0}{a_1} \tag{10}$$

For testing the methodology freshly cut apple slices which were drying with 38 ºC heated air are shown (Fig. 1). Since the initial moisture of the product reaches 90 % and the product unit is large enough, then the drying rate expression chooses parabolic relationship $K(t) = a_2 t^2 + a_1 t + a_0$, which could well describe the drying rate changes during the drying process (sample surface and internal moisture migration).

Fig. 1. **Fresh and dried apple slices during drying experiment**

Using the methodology described above and the experimental data obtained the drying rate expression for fresh cut apple slice layer drying with 38 ºC heated air :

$$K(t) = 1.2 \cdot 10^{-9} \cdot t^2 - 1.2 \cdot 10^{-6} \cdot t + 1.73 \cdot 10^{-3} \ , \qquad (11)$$

with the coefficient of determination $\eta^2 = 0.96$. There t-drying time (min).

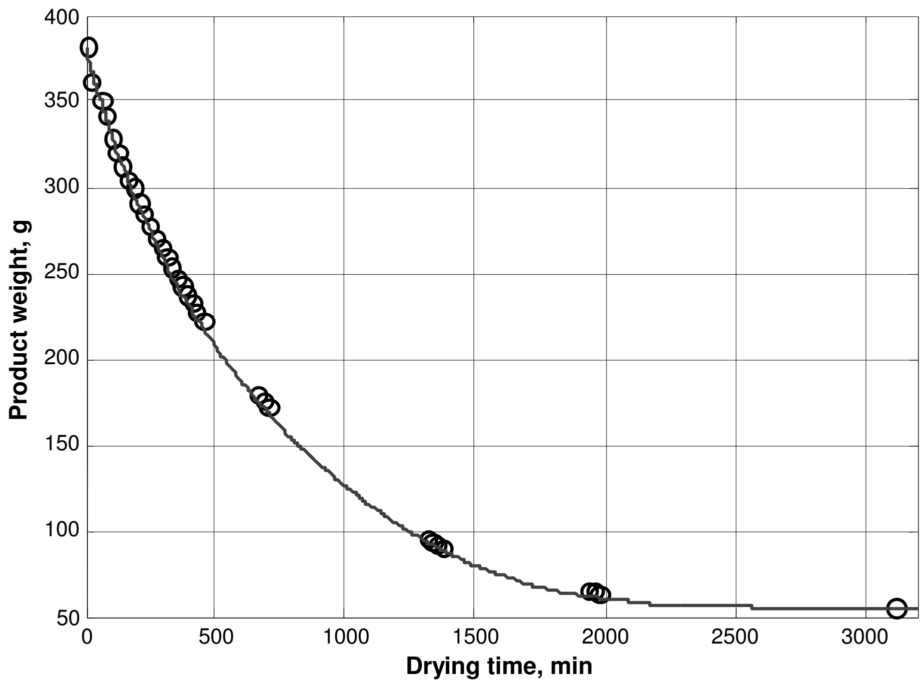

Fig. 2. **Apple slice weight changes during the drying process:**
ooo – experimental results; **----** – theoretical results

Putting (11) in problem (8) solution we received theoretical mass changes during the experiment and compared these results with the experimental data (Fig. 2). The max absolute value of the difference between the corresponding theoretical and experimental data was 8 grams with standard derivation 2 grams.

Conclusions

1. The proposed methodology is applicable for finding the porous material drying coefficient in a thin layer of material.
2. The developed mathematical model for thick layers of grain drying can be applied to other porous material layer drying process clarifying previously expression of the drying coefficient.
3. Theoretically the necessary drying time with constant conditions and linear drying rate is defined.
4. This methodology can be applied to find the drying rate of the material at different temperatures and combining the results to find the coefficient depending on both, the drying time and temperature.

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