MATHEMATICAL MODEL OF TRACTOR AGGREGATE

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Abstract. This paper presents a mathematical model of vertical oscillations of a tractor aggregate with an implement. A mathematical model of the tractor aggregate is derived to suit the control task of damping of oscillations. The dynamical model of wheels, coupled with the body, is representing the tractor with an implement. The wheels due to the compressibility of the tires are represented as spring-damper systems. The body of the tractor and implement are represented as a two beam system, connected with thse joint and spring-damper system. In order to derive a dynamic model of the system, the necessary calculations are made. The Lagrange's equation of second kind is used to derive the equations of motion of the described system.

Keywords: mathematical model, tractor aggregate oscillation.

Introduction

During transport operation the tractor aggregate with mounted heavy implement usually starts to oscillate travelling on uneven road surface. Vertical oscillations of the tractor aggregate reduce controllability of the front wheels and cause a necessity to reduce the transportation speed. The purpose of mathematical modeling of vertical oscillations of the tractor aggregate is to find out the optimal parameters of implement suspension system. Amplitudes of vertical oscillation of the wheel axis are used as criteria for parameter evaluation. The main task is to minimize these amplitudes. The mathematical model of the tractor aggregate with implement is a necessary step for simulation the oscillations during transport operation.

Materials and methods

A simple model representing the tractor aggregate with a mounted implement is derived using rigid body mechanics and illustrated in Fig. 1. The model of the real system is reduced to two dimensions only, which means that the lateral dynamics is neglected.

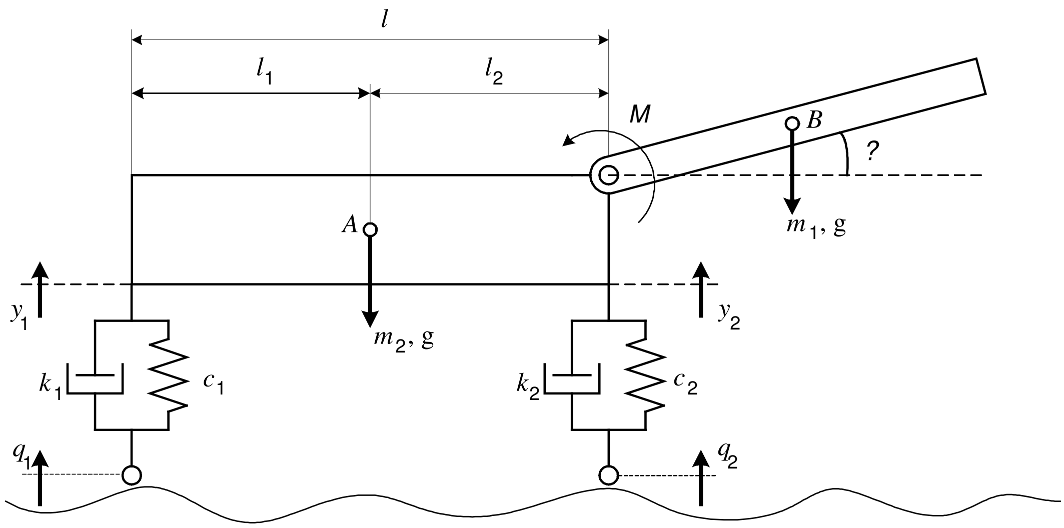

Fig. 1. **Mechanical model of tractor aggregate with mounted implement**

The dynamics of wheels due to the compressibility of the tires is modelled with spring and damper systems. The body of the tractor and implement are represented as a two beam system, connected with the joint and spring-damper. The road disturbances q_1 and q_2 act on the spring-damper systems of wheels and y_1 and y_2 correspond to displacements of the wheel axis. The torque M is generated on the revolution joint of implement connection to the tractor body with the hydraulic lift cylinder.

To simplify the forced oscillation further analysis, the following simplifications and conditions are introduced:

- tractor moving speed at straight road is constant;
- road surface under the tractor right and left sides is the same;
- tractor tires are on an independent contact with the road surface;
- road profile shown as sinusoidal function at a certain step of the road roughness;
- transmission rotation oscillations are neglected.

In order to derive a dynamic model of the system the necessary kinematic calculations are made. The position y_{m1} of the body mass m_1 is derived from the positions y_1 and y_2 (Fig. 2). The movement of the center of gravity of m_1 can be described with a position deviation y_{m1} and the angle β.

The position y_{m1} is found, if the horizontal movement of the tractor aggregate is neglected:

$$y_{m1} = \frac{l_1}{l} y_1 + \frac{l_2}{l} y_2 , \tag{1}$$

where y_{m1} – position of tractor mass center;
l_1 – distance from mass center to front tire;
l_2 – distance from mass center to rear tire;
l – wheel bases of tractor;
y_1 – vertical position of front wheels axis;
y_2 – vertical position of rear wheels axis.

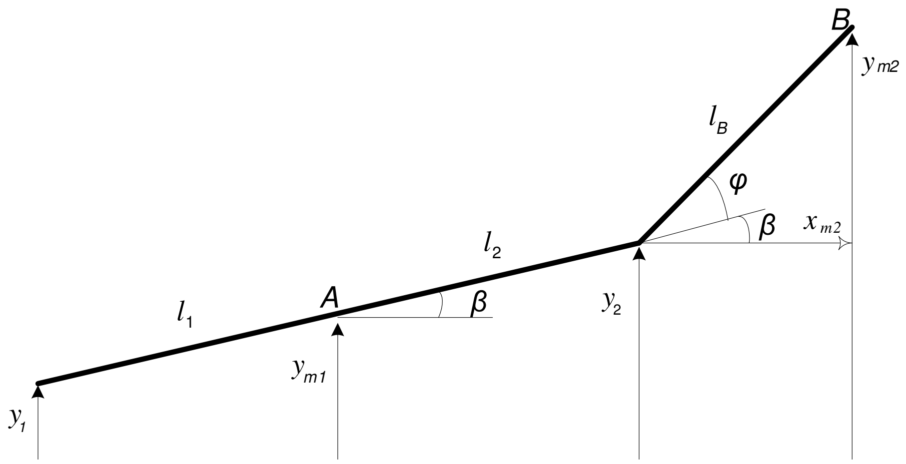

Fig. 2. **Kinematic of simple tractor aggregate mechanics**

The velocity of the tractor mass center A is therefore:

$$v_1 = \dot{y}_{m1} = \frac{l_1}{l} \dot{y}_1 + \frac{l_2}{l} \dot{y}_2 . \tag{2}$$

An expression for the angle β can be found from relation:

$$y_2 - y_1 = (l_2 + l_1) \cdot \sin \beta = l \cdot \sin \beta . \tag{3}$$

Since $\sin\beta \approx \beta$ for small angles the following expression is used for β.

$$\beta = \frac{1}{l} (y_2 - y_1) . \tag{4}$$

Then the corresponding angular velocity ω_1 is:

$$\omega_1 = \dot{\beta} = \frac{1}{l} (\dot{y}_2 - \dot{y}_1) . \tag{5}$$

The horizontal and vertical position of m_2 as shown in Fig. 2. is:

$$\begin{aligned} x_{m2} &= l_B \cdot \cos(\phi + \beta) \\ y_{m2} &= y_2 + l_B \cdot \sin(\phi + \beta) \end{aligned} . \tag{6}$$

The derivatives of the equations (6) give the following expression:

$$\begin{aligned} \dot{x}_{m2} &= -l_B \cdot \left(\dot{\varphi} + \dot{\beta}\right) \cdot \sin\left(\varphi - \beta\right) \\ \dot{y}_{m2} &= \dot{y}_2 + l_B \cdot \left(\dot{\varphi} + \dot{\beta}\right) \cdot \cos(\varphi + \beta) \end{aligned} . \tag{7}$$

Knowing the equation (7) derivatives the velocity of the mass center *B* of the implement by the following expression is obtained:

$$v_2 = \sqrt{\dot{x}_{m2}^2 + \dot{y}_{m2}^2} = \sqrt{l_B \cdot \left(\dot{\varphi} + \dot{\beta}\right) + \dot{y}_2^2 + 2 l_B \cdot \dot{y}_2 \cdot \left(\dot{\varphi} + \dot{\beta}\right) \cdot \cos\left(\varphi - \beta\right)} . \tag{8}$$

Then angular velocity is:

$$\omega_2 = \dot{\varphi} + \dot{\beta} . \tag{9}$$

Lagrange's equations of second kind [1; 2] are used to derive the equation of motion of the described system. In order to use the Lagrange equations the motion of the bodies of the system must be described with generalized coordinate *q*. In this case, the generalized coordinates are selected as to fully describe the system:

$$q = \begin{bmatrix} y_1 \\ y_2 \\ \varphi \end{bmatrix} .$$

Lagrange's equation defined as follow:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = Q .$$

where *L* is the Lagrangian which is defined as the difference of the kinetic energy *T* and the potential energy *U* of the entire system.

$$L(q, \dot{q}) = T(q, \dot{q}) - U(q) .$$

In this case, the vector *Q* represented as a general effect on the forces of the corresponding generalized coordinates:

$$Q = \begin{bmatrix} F_1 \\ F_2 \\ M \end{bmatrix} .$$

The dampers can not be modeled with Lagrange's equations of the second kind, so these are neglected, but are easily added afterwards to the resulting differential equations. The expression for the kinetic energy in the system is divided into one term of translational energy and one term of rotational energy. The potential and kinetic energy are similar to the record of the front wheel loader model [3]. The expression for the kinetic energy can be expressed as:

$$\begin{aligned} T &= \frac{1}{2} m_1 \cdot v_1^2 + \frac{1}{2} m_2 \cdot v_2^2 + \frac{1}{2} I_1 \cdot \omega_1^2 + \frac{1}{2} I_2 \cdot \omega_2^2 = \\ &= \frac{1}{2} m_2 \left(l_B^2 \left(\dot{\varphi} + \frac{1}{L}\left(\dot{y}_2 - \dot{y}_1\right)\right)^2 + \dot{y}_2^2 + 2 l_B \cdot \dot{y}_2 \left(\dot{\varphi} + \frac{1}{L}\left(\dot{y}_2 - \dot{y}_1\right)\right) \cos\left(\varphi + \frac{1}{L}\left(y_2 - y_1\right)\right)\right) + \\ &+ \frac{1}{2} m_1 \left(\frac{l_1}{l} \dot{y}_1 + \frac{l_2}{l} \dot{y}_2 \right)^2 + \frac{1}{2} I_1 \left(\dot{\varphi} + \frac{1}{l}\left(\dot{y}_2 - \dot{y}_1\right)\right)^2 + \frac{I_2}{2 l^2}\left(\dot{y}_2 - \dot{y}_1\right)^2 \end{aligned} \tag{10}$$

where J_1 – inertia of the tractor body;
J_2 – inertia of the implement.

The total potential energy of the system can be expressed as:

$$U = \frac{1}{2}c_1 \cdot y_1^2 + \frac{1}{2}c_2 \cdot y_2^2 + m_1 \cdot g \cdot y_{m1} + m_2 \cdot g \cdot y_{m2} =$$
$$= \frac{1}{2}c_1 \cdot y_1^2 + \frac{1}{2}c_2 \cdot y_2^2 + m_1 \cdot g\left(\frac{l_1}{l} y_1 + \frac{l_2}{l} y_2\right) + m_2 \cdot g\left(y_2 + l_B \sin\left(\varphi + \frac{1}{l}(y_2 - y_1)\right)\right) \tag{11}$$

The differential equation of generalized coordinate y_1 can be expressed as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_1}\right) - \frac{\partial L}{\partial y_1} = F_1$$
$$\left(m_1\left(\frac{l_B}{l}\right)^2 + m_2\left(\frac{l_2}{l}\right)^2 + \frac{I_1}{l^2} + \frac{I_2}{l^2}\right)\ddot{y}_1 +$$
$$+\left(-m_2\left(\frac{l_B}{l}\right)^2 - m_2\frac{l_B}{l}\cos\left(\varphi + \frac{1}{l}(y_2 - y_1)\right) + m_1\frac{l_1 \cdot l_2}{l^2} - \frac{I_1}{l^2} - \frac{I_2}{l^2}\right)\ddot{y}_2 + \tag{12}$$
$$+\left(-m_2\frac{l_B^2}{l} - \frac{I_1}{l}\right)\ddot{\varphi} + c_1 \cdot y_1 + m_1 \cdot g \cdot \frac{l_1}{l} - m_2 \cdot g \cdot \frac{l_B}{l}\cos\left(\varphi + \frac{1}{l}(y_2 - y_1)\right) = F_1 \quad .$$

Then the differential equation of the coordinate y_2 is:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{y}_2}\right) - \frac{\partial L}{\partial y_2} = F_2$$
$$\left(-m_2\left(\frac{l_B}{l}\right)^2 - m_2\frac{l_B}{l}\cos\left(\varphi + \frac{1}{l}(y_2 - y_1)\right) + m_1\frac{l_1 \cdot l_2}{l^2} - \frac{I_1}{l^2} - \frac{I_2}{l^2}\right)\ddot{y}_1 +$$
$$+\left(-m_2\left(\frac{l_B}{l}\right)^2 + 2m_1\frac{l_B}{l}\cos\left(\varphi + \frac{1}{l}(y_2 - y_1)\right) + m_2\left(\frac{l_2}{l}\right)^2 + \frac{I_1}{l^2} + \frac{I_2}{l^2} + m_2\right)\ddot{y}_2 +$$
$$+\left(-m_2\frac{l_B^2}{l} + m_2 \cdot l_B \cdot \cos\left(\varphi + \frac{1}{l}(y_2 - y_1)\right) + \frac{I_2}{l}\right)\ddot{\varphi} + m_2 \cdot l_B \cdot \sin\left(\varphi + \frac{1}{l}(y_2 - y_1)\right) \cdot \tag{13}$$
$$\cdot\left(\frac{\dot{y}_2}{l}\left(\dot{\varphi} + \frac{1}{l}(\dot{y}_2 - \dot{y}_1)\right) - \left(\dot{\varphi} + \frac{1}{l}(\dot{y}_2 - \dot{y}_1)\right)^2 - \frac{\dot{y}_2}{l^2}\right) \cdot$$
$$\cdot m_2 \cdot g \cdot \frac{l_B}{l}\cos\left(\varphi + \frac{1}{l}(y_2 - y_1)\right) + c_2 \cdot y_2 + m_1 \cdot g \cdot \frac{l_2}{l} + m_2 \cdot g = F_2 \quad .$$

And for the generalized coordinate φ:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\varphi}}\right) - \frac{\partial L}{\partial \varphi} = M$$
$$\left(-m_2 \cdot \frac{l_B^2}{l} - \frac{I_2}{l}\right)\ddot{y}_1 + \left(m_2 \cdot \frac{l_B^2}{l} + m_2 \cdot l_B \cdot \cos\left(\varphi + \frac{1}{l}(y_2 - y_1)\right) + \frac{I_2}{l^2}\right)\ddot{y}_2 +$$
$$+\left(-m_2 \cdot l_B^2 - I_2\right)\ddot{\varphi} + m_2 \cdot g \cdot l_B \cdot \cos\left(\varphi + \frac{1}{l}(y_2 - y_1)\right) = M \quad .$$

Results and discussion

The applied forces F_1 and F_2 depend on the tractor aggregate speed, roughness height, its character and wheel stiffness, and damping characteristics. In order to verify the theoretical conclusions in certain circumstances, the forces F_1 and F_2 express the tractor aggregate running on an artificial roughness road (Figure 4).

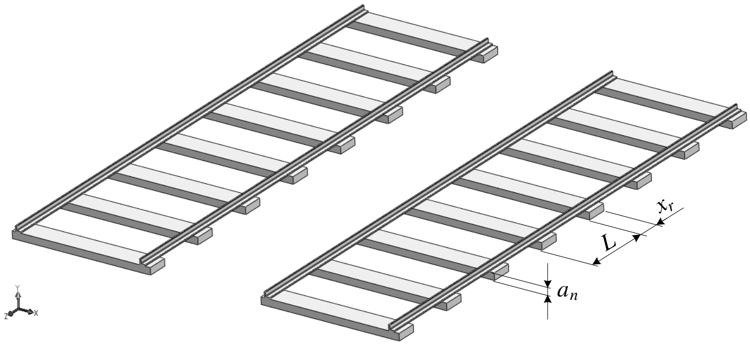

Fig. 4. **Scheme of artificial roughness road**

The front wheel applied forces of the tractor can be expressed by the following expression:

$$F_1 = k_{r1} \cdot q_1 + c_{r1} \cdot \dot{q}_1 . \quad (15)$$

But for the rear wheels as:

$$F_2 = k_{r2} \cdot q_2 + c_{r2} \cdot \dot{q}_2 , \quad (16)$$

where k_{r1} and k_{r2} – stiffness coefficients of tractor front and rear wheels, kN·s·m^{-1};
c_{r1} and c_{r2} – damping coefficients of tractor front and rear wheels, kN·m^{-1};
q_1 and q_2 – vertical displacement from road disturbances to front and rear wheels, m;
$\dot{q}_1$ and $\dot{q}_2$ – vertical velocity from road disturbances to front and rear wheels, m·s^{-1}.

The tractor wheel motion on artificial roughness is described as a sinusoidal disturbance:

$$q = a_n \cdot \sin(\omega \cdot t) , \quad (17)$$

where a_n – road roughness height, m;
ω – angular frequency, s^{-1};
t – time, s.

Sinusoidal disturbance is recommended to use [4; 5] for investigation of running smoothness of tractors. Vertical velocity from road disturbances:

$$\dot{q} = a_n \cdot \omega \cdot \cos(\omega \cdot t) . \quad (18)$$

Using the equation (17) and (18), the tire stiffness and damping coefficients the forces F_1, F_2 can be found with equations (15) and (16). Inserting the forces F_1, F_2 and different M values with the stiffness coefficient in equations (12), (13) and (14) the mathematical model for tractor aggregate vertical oscillations is obtained.

Conclusions

1. The applied forces F1 and F2 depend on the tractor aggregate speed, roughness height, its character and wheel stiffness, and damping characteristics.
2. If the tractor wheel motion on artificial roughness is described as a sinusoidal disturbance, the tire stiffness and damping coefficients, the forces F1, F2 can be found.
3. Inserting the forces F1, F2 and different M values with the stiffness coefficient in Lagrange equations the mathematical model for tractor aggregate vertical oscillations is obtained.

Acknowledgement

The paper is written by financial support of European Structural Fund – Support for Realization of Doctoral Studies in the Latvia University of Agriculture – realized by the Project Department of the Latvia University of Agriculture (contract no. 2009/0180/1DP/1.1.2.1.2/09/IPIA/VIAA/017).

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