DEFINITION OF POISSON'S RATIO OF ELASTOMERS

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Abstract. Design with rubber (elastomer) – metal elements (futter – RME) is successfully used for the construction machine. Due to the continual development of machinery and equipment, they are ever increasing in intensity and dynamic load requirements of the components of the relative position accuracy. The relative position variation (such as radial, sharp and angular displacement transmission shafts, bearings sitting inaccuracies, delays threaded plugs, etc.) may result from deficiencies in the assembly, temperature changes and deformation forces, process constraints. To analyze the RME construction work there is a need to know elastomer mechanical modules. In work it is offered to define the factor of Poisson's ratio from experiments on axial compression of multilayered packages from thin-layer elastomer layer. To exclude influence of ruggedness of experimental installation on the indication of the devices, it is offered to select a number of layers in a package and geometrical parameters elastomer layers so that deposit of a layer at the expense of the change of its form and compressibility elastomer, were one order. For processing of the experimental data it is offered to use analytical decisions ("force-settlement") for a package from thin elastomer layers. The calculation example confirms applicability of the considered technique for definition of the factor of Poisson's ratio for elastomers.

Key words: compressive stiffness, rubber, Poisson's ratio, force-settlement.

Introduction

Over the past 20-30 years, using the specific properties of rubber (elastomer) (high elasticity, resistance to environmental influences, good dynamic performance, low compressibility, almost linear relationship between stress and strain at strains up to 15÷20 %) shock absorbers were developed. For efficient utilization of rubber elements in modern mechanical engineering, including vibration insulation, it is necessary to be able to calculate the characteristics of compressive stiffness of rubber - metal elements (futter RME). For poorly compressed elastomers (rubbers) experimental definition of the value of the factor of Poisson's ratio, as even the small error of this factor leads to essential errors at analytical calculation deposits thin-layer elastomer elements, is problematic. In the work it is offered to define the factor of Poisson's ratio from experiments on axial compression of multilayered packages from thin-layer elastomer layer. To exclude the influence of ruggedness of experimental installation on the indication of devices, it is offered to select a number of layers in a package and geometrical parameters elastomer layers so that deposit of a layer at the expense of the change of its form and compressibility elastomer, were one order. For processing of the experimental data it is offered to use analytical decisions ("force-settlement") for a package from thin elastomer layers. The calculation example confirms applicability of the considered technique for definition of the factor of Poisson's ratio for elastomers.

Manufacture of high precision machinery compensatory RME may be required for very high: a high rigidity and load carrying capacity of the basic direction of loading and at the same time low resistance to displacements in the direction of compensation, to be able to compensate for even the smallest displacement and vibration.

Elastomer mechanical modules

Elastomer share of the operating compression determines the importance of the landed value of the force acting on the elastomer compression direction. Usually in calculations elastomer weak compressibility effects (Poisson's ratio) are not taken into account and the elastomer is characterized only by the shear modulus G or the elastic modulus E. If, $\mu \approx 0.5$ then, $E = 2(1 + \mu)G \approx 3G$ and elastomer aspect ratio error of the low compressibility of non-compliance with the balance of perched is far less, if the resulting mainly perched depends on elastomers in shape. If the elastomeric elements are plans ($a / h > 10$, where: a, h – respectively, an elastomer layer width and height), then the power – sat down in an analytical dependence on analytical calculation is necessary to take into account the elastomeric layer of weak compressibility effects, which depend on the Poisson's ratio μ values. To

calculate such elastomers it is necessary to employ the module of volume compression.
$$K = \frac{E}{3(1-2\mu)}$$

If the Poisson's ratio μ is close to 0.5, the spatial compression modulus K is strongly dependent on the Poisson's ratio values of accuracy, but the modulus of elasticity E changes very little. This is shown in Table 1.

Table 1

Poisson's ratio	**Modulus of elasticity**	**Spatial compression modulus**
$\mu = 0.45 \pm 1$ % = 0.45450÷0.44550	$E^* = (1.0031 \div 0.9968)\ E$	$K^* = (1.0992 \div 0.9168)\ K$
$\mu = 0.49 \pm 1$ % = 0.49490÷0.48510	$E^* = (1.0033 \div 0.9967)\ E$	$K^* = (1.9608 \div 0.5100)\ K$
$\mu = 0.495 \pm 1$ % = 0.49995÷0.49005	$E^* = (1.0033 \div 0.9934)\ E$	$K^* = (100 \div 0.5025)\ K$

From Table 1 it follows that if the Poisson's ratio values are approaching 0.5, there is a very precise set factor value. Currently there is no such method, which allows so precisely to determine the Poisson's ratio.

Possible variants of experiments

Elastomer element perched below the load can be conditionally divided into two parts:

$$\Delta = \Delta_F + \Delta_S \tag{1}$$

where Δ_F – elastomeric element landed the part which is related to its shape (mostly sat down when the RME geometric parameter $a / h < 2 \div 5$), – elastomeric element landed the part which is related to its compressibility (mostly sat down when the RME geometric parameter $a / h >> 10$).

Δ_S – elastomeric element's settlement that is associated with a change of the elastomer volume.

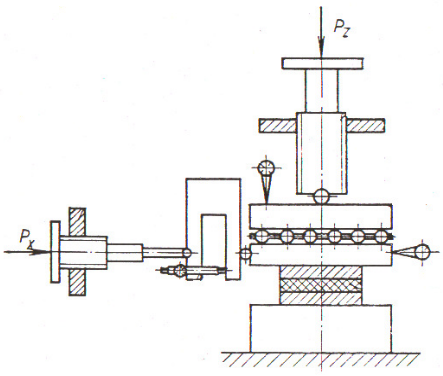
In order to accurately determine the Poisson's ratio, it is necessary to define precisely the elastomeric element perched share, which is related to the compressibility. In theory, two methods can be used:

1. take a very thin elastomeric element, which perched part is much less than the sat down in part;
2. use the volume compression experiments to delete the perched share.

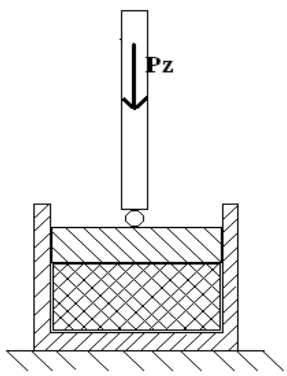
Both these options would not give accurate results because:

The first version, which has taken a very thin elastomeric element, needs very sophisticated equipment and precision measurement (see Fig. 1, a), the elastomeric elements of the plan hardness are close to the experimental equipment and material hardness can reach 25 kgs·mm^{-2}. It would be difficult to divide, where the elastomeric element deformation is and deformation of the equipment;

The second version, the elastomers are limited in shape (see Fig.1, b.), form the same problems.



a)



b)

Fig. 1. **Possible variants of experiments**

Recommendable variants of experiment

To remove the maximum hardness of the pilot plant, it is proposed to use experiments with multi-layer package of plans to sharp compression of the layers of elastomer. Followed to the recommend such elastomeric layer dimensions (a, b and h), the elastomer component parts and landed value were one order. With such techniques, principally the pilot scheme is given in Fig. 2.

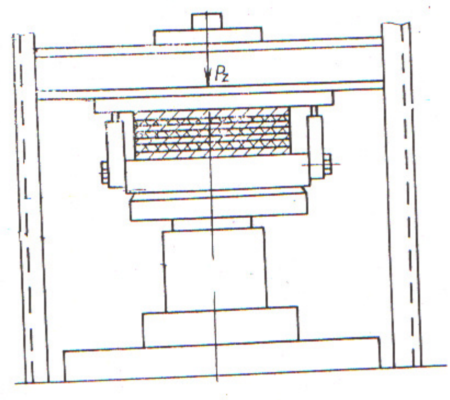


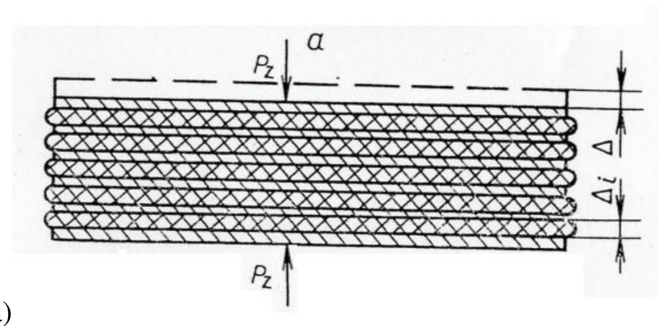
Fig. 2. **Recommendable variants of experiment**

To analyze the experimental data (calculated Poisson's ratio μ value), we use the known analytical solution "force (P_z) – settlement (Δ)" multi-layer rubber – metal element to the central axis compression.

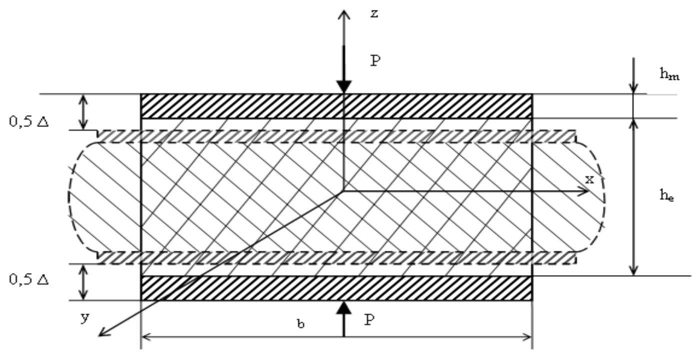
Materials and methods

The method of obtaining analytical dependence "force-settlement" of multi-layer compensating element under axial compression is considered. Application of the proposed method is shown by the example of designing a multi-layer shock-absorber, which consists of thin flat rectangular elements.

The geometrical design model is shown in Figure 3 (a) and (b).



a)



b)

Fig. 3. **Model of calculation:** a – shock absorber; b – n^{th}-layer of shock absorber

The proposed method is using the variational method, which is based on the principle of minimal potential energy [1] for low compressible materials. The potential energy of the studied element in case of small deformations is written this way:

$$J = \sum_{i=1}^{i=n} G_i \int_V \begin{bmatrix} (\varepsilon^2{}_{xx} + \varepsilon^2{}_{yy} + \varepsilon^2{}_{zz} + 2(\varepsilon^2{}_{xz} + \varepsilon^2{}_{yz} + \varepsilon^2{}_{xy}) + \\ + \dfrac{3\mu_i}{1+\mu_i} S_i(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}) - \dfrac{9\mu_i(1-2\mu_i)}{4(1+\mu_i)^2} S_i^2 \end{bmatrix} \cdot dV - P\Delta \cdot G, \tag{2}$$

where G_i – modulus of rigidity for each layer;
μ_i – Poisson's ratio of material of each rubber layer;
P – longitudinal force of compression;
Δ – settlement of entire element;
s_i – hydrostatic pressure function in each layer;
u, v, w – displacements of randomly chosen points in each layer of the layer, respectively, in directions x, y, z;
V – volume of each layer. The summing up is carried out for all rubber and non-elastomeric layers of a multi-layer element.

Deformations ε_{ij} in each layer are found using formulae:

$$\varepsilon_{xx} = \frac{du}{dx};\ \varepsilon_{yy} = \frac{dv}{dy};\ \varepsilon_{zz} = \frac{dw}{dz};$$

$$\varepsilon_{xy} = \frac{1}{2}(\frac{du}{dy} + \frac{dv}{dx});\ \varepsilon_{yz} = \frac{1}{2}(\frac{dv}{dz} + \frac{dw}{dy});\ \varepsilon_{zx} = \frac{1}{2}(\frac{du}{dz} + \frac{dw}{dx}). \tag{3}$$

The potential energy for the entire element is calculated by summing up formula (2) for all rubber and non-elastomeric layers. Physical and mechanical features of the material layers and geometrical parameters of layers have such indexes: e – for rubber layers; m – for non-elastomeric layers.

In order to use functional (2) when choosing functions of displacements (x, y, z), $v(x, y, z)$, $w(x, y, z)$ and functions of hydrostatic pressure $s(x, y, z)$, it is enough to fulfil geometrical boundary conditions and the conditions of coupling rubber and non-elastomeric layers for displacement functions. For simplicity let us suppose that all the layers have the same dimensions in the design (a and b), all the rubber layers have the width h_e, and the non-elastomeric layers have the width h_m. For the considered problem the necessary geometrical conditions are:

$$w_e(x, y, 0.5h_e) = -0.5\Delta;\ w_e(x, y, -0.5h_e) = 0.5\Delta$$

$$u_e(x, y, \pm 0.5h_e) = u_m(x, y, \pm 0.5h_e);\ v_e(x, y, \pm 0.5h_e) = v_m(x, y, \pm 0.5h_e) \tag{4}$$

When writing displacement functions analytically let us suppose that: for rubber layers the hypothesis of plane sections is true; for non-elastomeric layers the condition of homogeneous deformation is fulfilled. In this case, taking into account the geometrical conditions (4), the desired displacement functions can be chosen in the form for:

- rubber layers:

$$u_e = C_1x(z^2 - h_e^2/4) + K_1x;\ v_e = C_2\,y(z^2 - h_e^2/4) + K_2y;$$

$$w_e = -C_3(z^3/3 - zh_e^2/4)/h_e^3 - C_4z;\ s_e = C_5(z^2 - \mathrm{h_e}^2/4); \tag{5}$$

- non-elastomeric layers:

$$u_m = K_1x,\ V_m = K_2\,y,\ w_m = s_m = 0,\ w_m = s_m = 0. \tag{6}$$

C_1, C_2, C_3, C_1, C_2, K_1, K_2 – are unknown constants, which can be found using the settlement of the element Δ from the minimum condition of full potential energy of deformation (2) of the entire element:

$$\frac{\partial J(C_1, C_2, C_3, C_4, K_1, K_2)}{\partial(C_1, C_2, C_3, C_4, K_1, K)} = 0, \tag{7}$$

where Δ – desired unknown settlement of the element, which, by using equations (4)-(7), can be found from the equation:

$$\Delta = -C_3 h_e^3/6 + C_4 h_e. \tag{8}$$

From algebraic equation system (6) and (7) for the considered element the desired dependence "force-settlement" can be written as:

$$\Delta = \frac{P\,h_e n}{2{,}5\,G_e ab} \frac{1 + 1.25\dfrac{B_1 B_2}{\chi(B_1 + B_2)}}{1 + \dfrac{B_1\,B_2}{B_1 + B_2 + \dfrac{1-2\mu}{\mu} B_1 B_2}}, \tag{9}$$

where:

$$B_1 = 1 + \frac{5\,\alpha^2}{12};\ \ B_2 = 1 + \frac{5\,\beta^2}{12}$$
$$\alpha = \frac{a}{h_e},\quad \beta = \frac{b}{h_e},\ \chi = \frac{G_m h_m}{G_e h_e}, \tag{10}$$

where a, b, h_e, h_m – are geometrical parameters of flat rectangular rubber and non-elastomeric layers;
G_e, G_m – modulus of rigidity of material, respectively, of rubber and non-elastomeric layers;
n – number of rubber layers in the packet.

If rubber and non-elastomeric layers have different dimensions, which let us ignore the low compressibility of rubber layers and flexibility ($h_e < h_m$, $G_e << G_m$, t.i. parameter $\chi \to \infty$) of non-elastomeric layers, then from formula (7) we obtain the dependence for element settlement:

$$\Delta_0 = \frac{Ph\,n}{2.5\,G\,ab} \frac{1}{1 + \dfrac{\alpha^2 \beta^2}{\alpha^2 + \beta^2}}, \tag{11}$$

which coincide with the dependence "force-settlement", obtained in the work [3] without taking into account the compressibility of rubber and deformation of non-elastomeric layers.

The authors in work [3] show the dependence "force-settlement" results of RME, which is derived from a rectangular cell (a = b = 40 mm, h_e = 0.2 cm, G_e = 4.5 kg·cm^{-2}, h_m = 0.02 cm, G_m = 2.8·10^5 kg·cm^{-2}) central axis compression. Table 3 gives the Poisson's ratio results from the formula (9).

Table 3

Poisson's ratio values

Rubber grade	**Rubber shear modulus *G*, kgs·cm^{-2}**	**Poisson's ratio, μ**
8871	4.5	0.4981
8470	8.0	0.4931
ИПР-1124	16.0	0.4903

The obtained results are verified, calculating the dependence "force-settlement", for RME with other geometric dimensions and comparing with the experimental results.

Elastomeric elements:

$a = b = 8$ cm, $h_e = 0.2$ cm, $G_e = 4.5$ kg·cm^{-2}, $h_m = 0.02$ cm, $G_m = 2.8\cdot 10^5$ kg·cm^{-2}, $P = 4000$ kg

The dependence "force-settlement" is calculated with formula (9), and is compared with the experimental data of the work [3]. $\Delta_{exp} = 1.41\cdot 10^{-3}$ cm, which coincides well enough to calculate landed from formulas (9) – $\Delta_{calc} = 1.38\cdot 10^{-3}$cm. The obtained result is in good agreement with the experiments.

Conclusions

The issue in methodology makes it possible, through multi-layer planning RME packet sharp compressive loads, with sufficient precision of the Poisson's ratio, which is necessary to know, to analyze and to design effective rubber – metal elements with thin elastomer layers. The methodology takes into account deformation of not elastomeric layer that increases exactness. Having the experimental data from Δ_{exp} formula (9) or from formula (11) if it is possible to ignore deformation of not elastomeric layer, it is easy to get the values of Poison's ratio.

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