MATHEMATICAL SIMULATION OF BEET VIBRATIONAL EXTRACTION

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Abstract. A new mathematical model is developed which describes the process of direct sugar beet extraction from soil, carried out by interaction of the vertical impact force and the tractive effort, which are transmitted to the root crop from the vibratory digging tool. Sets of differential equations have been obtained the solution of which enables to determine the law of the root crop movement during their direct vibrational extraction.

Keywords: root crops, vibrational extraction, digging tool, frequency.

Introduction

The use of sugar beet vibrational extraction from the soil has a number of essential advantages in comparison with other ways. It is characterized by lesser damage of the roots, lower losses of the crop during the harvest period, more intensive cleaning of the root crops from the clods of soil, lesser blocking up of the working channel of the digger with soil and plant residues. Therefore, this technological process requires detailed analytical research, further development, and manufacturing application of advanced vibratory digging tools.

Research results

At first let us carry out the necessary formalization of the technological process to be considered. Despite the fact that the process of sugar beet extraction from the soil takes a short interval of time (the forward speed of the root diggers can reach 2 $m \cdot s^{-1}$), all the process can be conditionally divided into separate interconnected successive operations [1]. As it was noted above, extraction is possible only in the case of a symmetric grip of the root crop by the digging tool with simultaneous transformation of the vibrations of the root crop into its angular vibrations around a conditional point of fixing of the root crop in soil.

At the first stage of extraction, and especially at the first vibrations, the restoration force at the angular vibrations and therefore, its moment in relation to the conditional point of fixing will be maximal. That is why the angle of inclination of the root crop will be sufficiently small and full (or partial) restoration of its vertical position owing to the forward movement of the digger will be possible. Nevertheless, due to the action of the root crop, the forward vibrations together with the soil surrounding it, the compactness of the soil will decrease, and the restoration force at the angular vibrations will decrease, too. So, with each successive vibration the angle of inclination of the root crop will increase, and restoration of the previous position will decrease. The root crop will be loosened around the conditional point of its fixing with gradual increase of its inclination angle forward in relation to the course of the digger. This will lead to the loss of the contact between the root crop and the loose soil in the direction of the movement of the digger, beginning from the top part of the root crop conic surface, gradually approaching its conditional point of fixing in soil. So, as it was stated above, it follows that the destruction of the root crop contact with soil occurs simultaneously in two directions – along the forward movement of the digger and in the perpendicular direction (along the full depth of the root crop position in soil). Thus, the forces of root crop contacts with soil and the elasticity forces of soil will gradually decrease to such a minimal value at which the vibrational processes are transformed into the processes of root crop continuous movement upwards and forward – along the movement of the digger, and also into continuous root crop rotation around their centers of mass. The elasticity forces of the soil will pass into the resistance forces of the loose soil during the root crop movement through the working channel of the digger. After that the stage of the sugar beet direct extraction from the soil starts.

In order to develop a mathematical model, first of all we will make an equivalent scheme of the root crop interaction with the working surfaces of the vibratory digging tool during the root crop direct extraction (Figure 1). Let us present the vibratory digging tool in the form of two coupled digging surfaces (wedges) $A_1B_1C_1$ and $A_2B_2C_2$, each of which having an inclination in space at angles α, β, γ

and which are so located in relation to each other that a working channel is formed with its rear part narrowed. The pointed wedges produce vibrational movements in the longitudinal-vertical plane (the mechanism of the vibratory movement of the shares is not shown) with corresponding amplitudes and frequencies. The direction of the forward movement of the vibratory digging tool is shown by an arrow. The projections of the points B_1 and B_2 on the axis $O_1\gamma_1$ are marked accordingly by the points D_1 and D_2.

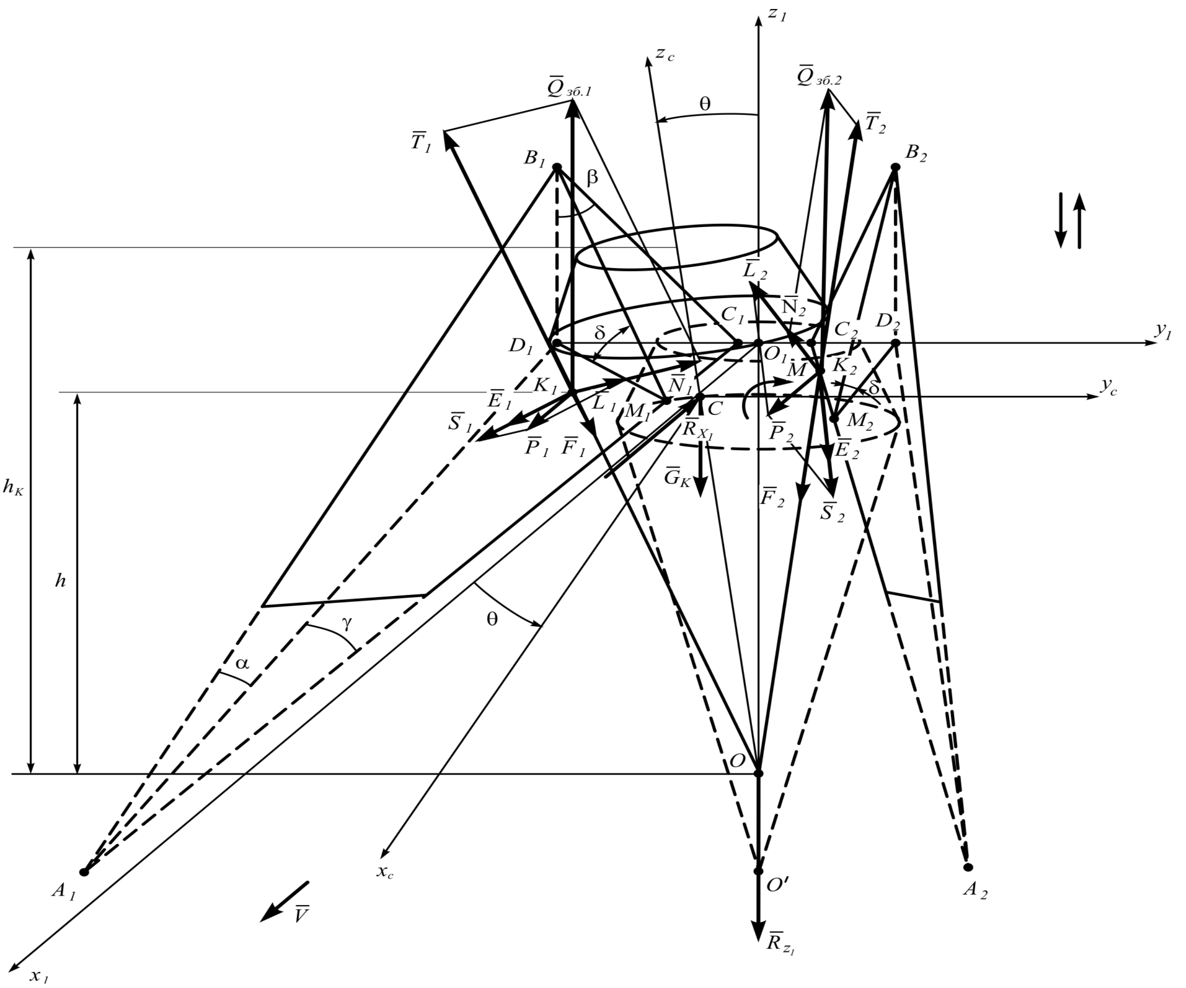

Fig. 1. **The equivalent scheme of root crop vibrational extraction from soil**

Let us consider how the root crop interacts with the surfaces of the wedges $A_1B_1C_1$ and $A_2B_2C_2$ in the corresponding points. The root crop is approximated by a cone-shaped body, and gripping of the root crop by the digging tool occurs symmetrically from its both sides. Let us assume further that the working surface of the wedge $A_1B_1C_1$ forms a direct contact with the root crop at the point K_1, and the surface $A_2B_2C_2$ – at the point K_2. The lines pass through the points K_1 and K_2 of the root crop contact, and the points B_1 and B_2 form a section with the sides of the wedges A_1C_1 and A_2C_2 corresponding to the points M_1 and M_2. Thus, δ is a dihedral angle ($B_1M_1D_1$) between the lower base of $A_1D_1C_1$ and the working surface of the wedge $A_1B_1C_1$, or, accordingly, a dihedral angle ($B_2M_2D_2$) between the lower base of $A_2B_2C_2$ and the working surface of the wedge $A_2B_2C_2$. Let us show the forces which arise owing to the interaction of the root crop with the vibratory digging tool. Interaction of the vibratory digging tool with the vertical impact force $\bar{Q}_{tr}$ proceeds according to the harmonious law of the form:

$$Q_{tr} = H \sin \omega t , \tag{1}$$

where H – amplitude of the impact force;
ω – frequency of the impact force.

This force plays a basic role during the loosening of soil in the zone of the digger working channel, and the root crop extraction. A specific impact force $\bar{Q}_{tr}$ is applied to the root crop from its two sides, and on the scheme it is presented by two components $\overline{Q}_{tr.1}$ and $\overline{Q}_{tr.2}$. These forces are applied accordingly to the points K_1 and K_2 at the distance h from the conditional point of fixing O, causing vibrations of the root crop in longitudinal-vertical planes which destroy the contact of the root crop with soil and create the required conditions for the extraction of root crops from the soil.

As the grips of the root crops are symmetric, it is obvious that there will be the following correlation:

$$Q_{tr.1} = Q_{tr.2} = \frac{1}{2} H \sin \omega t \cdot \tag{2}$$

Let us decompose the given forces into normals $\overline{N}_1$ and $\overline{N}_2$ and tangential components $\overline{T}_1$ and $\overline{T}_2$, as it is shown in the figure. As the vibratory digger moves forward in the direction of the axis O_1x_1 in relation to the root crop which is fixed in the soil, and during the moment of the grip of the root crop by the digging tool – in the direction of the axis O_1x_1, the motive forces $\overline{P}_1$ and $\overline{P}_2$ operate too. Let us decompose the forces $\overline{P}_1$ and $\overline{P}_2$ into two components: normals $\overline{L}_1$ and $\overline{L}_2$, and tangents $\overline{S}_1$ and $\overline{S}_2$ to surfaces $A_1B_1C_1$ and $A_2B_2C_2$, accordingly. Besides, at the points of contact K_1 and K_2 the forces of friction $\overline{F}_{K1}$ and $\overline{F}_{K2}$ act accordingly, which counteract the root crop sliding along the working surface of the wedges $A_1B_1C_1$ and $A_2B_2C_2$ during its grip by the vibratory digging tool. The vectors of these forces are directed opposite to the vector of the relative speed of the root crop sliding along the surface of the wedges. The root crop sliding along the surface of the wedges can move in the direction of forces $\overline{T}_1$, $\overline{T}_2$ (parallel lines B_1M_1 and B_2M_2) and in the direction, opposite to the action of forces $\overline{S}_1$, $\overline{S}_2$, due to the motion resistance force of the soil.

The vector of the relative speed of the root crop sliding along the surface of the wedges can be decomposed into components in the directions specified above. So, the force of friction $\overline{F}_{K1}$ can also be decomposed into two components: $\overline{F}_1$ in a direction, opposite to the vector $\overline{T}_1$, and $\overline{E}_1$ – in the direction of the vector $\overline{S}_1$. Similarly, the force of friction $\overline{F}_{K2}$ can be decomposed into two components: $\overline{F}_2$ – in a direction, opposite to the vector $\overline{T}_2$, and $\overline{E}_2$ – in the direction of the vector $\overline{S}_2$. It is obvious that F_1=F_2, E_1=E_2. In the center of the root crop mass (point C) the force of the root crop mass operates $\overline{G}_K$. The forces of resistance of the loose soil during the root crop movement through the working channel of the digger in the direction of axes O_1x_1 and O_1z_1 are designated as $\overline{R}_{x1}$ and $\overline{R}_{z1}$ accordingly.

During the direct root crop extraction from soil the rotation of the root crop around its center of mass (point C) will be carried out under the action of a pair of resistance forces of the loosened soil. We shall designate the moment of this pair of forces as M.

At the direct root crop extraction it is possible to consider the forces of resistance of the loosened soil depending on the speed of the root crop movement in the loosened soil or as a first approximation – as simple constants. Therefore, to simplify the mathematical model, we shall consider the forces $\overline{R}_{x1}$, $\overline{R}_{z1}$ and the moment of the pair M, as constants.

At first let us make differential equations for the movement of the center of the root crop mass (point C), i.e. the forward movement of the root crop along the axes O_1x_1 and O_1z_1. Considering the scheme of forces given above, the differential equation of the movement of the centre of the root crop mass in the vector form during their direct extraction will have the form:

$$m_k\overline{a} = \overline{N}_1 + \overline{N}_2 + \overline{L}_1 + \overline{L}_2 + \overline{F}_1 + \overline{F}_2 + \overline{E}_1 + \overline{E}_2 + \overline{G}_k + \overline{R}_{z1} + \overline{R}_{x1}, \tag{3}$$

where $\overline{a}$ – acceleration of the movement of the root crop mass center.

Since the process of extraction, as it has been specified above, occurs by symmetric gripping of the root crop by means of the digging tool, the root crop movement along the working channel of the digger occurs actually in longitudinal-vertical planes (planes $x_1O_1z_1$). This is why the vector equation (3) is reduced to a set of two equations in the projections to the axes Ox_1 and Oz_1.

After the definition of the values of all the forces which enter the vector equation (3), and their projections to the axes Ox_1 and Oz_1 we will produce the following two sets of differential equations:

$$\left.\begin{aligned}
&\ddot{x}_1 = \frac{1}{m_k}\left[\frac{\cos\delta\, tg\gamma}{\sqrt{tg^2\gamma + 1 + tg^2\beta}} + f\cos^2\delta\,\sin\left(\gamma + \frac{\alpha_{K1\max}}{2}\right)\sin\gamma + \right.\\
&\left. + f\cos\delta\cos\left(\gamma + \frac{\alpha_{K1\max}}{2}\right)\cos\gamma\right] H\sin\omega t + \frac{2}{m_k}\times\\
&\times\left[\frac{\sin\gamma\, tg\gamma}{\sqrt{tg^2\gamma + 1 + tg^2\beta}} + f\sin^2\gamma\,\sin\left(\gamma + \frac{\alpha_{K1\max}}{2}\right)\cos\delta + \right.\\
&\left. + f\sin\gamma\,\cos\gamma\,\cos\left(\gamma + \frac{\alpha_{K1\max}}{2}\right)\right] P_1 - \frac{R_{x1}}{m_k},\\
&\ddot{z}_1 = \frac{1}{m_k}\left[\frac{\cos\delta\, tg\beta}{\sqrt{tg^2\gamma + 1 + tg^2\beta}} - f\cos\delta\,\sin\left(\gamma + \frac{\alpha_{K_1\max}}{2}\right)\sin\delta\right] H\sin\omega t +\\
&\frac{2}{m_k}\left[\frac{\sin\gamma\, tg\beta}{\sqrt{tg^2\gamma + 1 + tg^2\beta}} - f\sin\gamma\,\sin\left(\gamma + \frac{\alpha_{K_1\max}}{2}\right)\sin\delta\right] P_1 - \frac{R_{z_1}}{m_k} - g,\\
&\omega t \in [2k\pi, 2(k+1)\pi],\; k = 0, 1, 2,...
\end{aligned}\right\} \tag{4}$$

$$\left.\begin{aligned}
&m_k\ddot{x}_1 = \frac{2P_1\sin\gamma\, tg\gamma}{\sqrt{tg^2\gamma + 1 + tg^2\beta}} + 2fP_1\sin^3\gamma\cos\delta + fP_1\sin 2\gamma\cos\gamma - R_{x1},\\
&m_k\ddot{z}_1 = \frac{2P_1\sin\gamma\, tg\beta}{\sqrt{tg^2\gamma + 1 + tg^2\beta}} - 2fP_1\sin^2\gamma\sin\delta - G_k - R_{z1},
\end{aligned}\right\} \tag{5}$$

$$\omega t \in [(2k-1)\pi,\; 2k\pi,\;],\; k = 1, 2,...$$

Thus the set of differential equations (4) describes the process of direct vibrational extraction of the root crops from soil (i.e. the distance at which the periodic impact force acts upon the root crop), and the set of differential equations (5) describes the process of the root crop extraction from soil when the impact force does not act upon it, i.e. the same vibratory digging tool can realize in different time intervals the process of the root crop digging as the usual share digger.

Let us solve the obtained sets of differential equations.

For the given sets of differential equations (4), (5) the initial conditions at $t = 0$ will be the following:

$$\dot{x}_1 = 0, \qquad \dot{z}_1 = 0 \tag{6}$$

$$x_1 = x_{10}, \qquad z_1 = -\frac{1}{3}h_k \tag{7}$$

The set of differential equations (4) is a set of linear differential equations of the second order. As it is known, it is solved in quadratures. For the simplification of recording of the set of differential equations (4), let us write:

$$\begin{aligned}
&\frac{1}{m_k}\left[\frac{\cos\delta\, tg\gamma}{\sqrt{tg^2\gamma + 1 + tg^2\beta}} + f\cos^2\delta\sin\left(\gamma + \frac{\alpha_{K1\max}}{2}\right)\sin\gamma + \right.\\
&\left. + f\cos\delta\cos\left(\gamma + \frac{\alpha_{K1\max}}{2}\right)\cos\gamma\right] = \phi_1,
\end{aligned} \tag{8}$$

$$\frac{2}{m_k}\left[\frac{\sin\gamma\, tg\gamma}{\sqrt{tg^2\gamma+1+tg^2\beta}}+f\sin^2\gamma\sin\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\cos\delta+ \right. \\ \left. + f\sin\gamma\cos\gamma\cos\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\right]=\psi_1, \tag{9}$$

$$\frac{1}{m_k}\left[\frac{\cos\delta\, tg\beta}{\sqrt{tg^2\gamma+1+tg^2\beta}}-f\cos\delta\sin\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\sin\delta\right]=\phi_2, \tag{10}$$

$$\frac{2}{m_k}\left[\frac{\sin\gamma\, tg\beta}{\sqrt{tg^2\gamma+1+tg^2\beta}}-f\sin\gamma\sin\left(\gamma+\frac{\alpha_{K1\max}}{2}\right)\sin\delta\right]=\psi_2. \tag{11}$$

Considering the expressions (8) – (11), the set of differential equations (4) will obtain the form:

$$\left.\begin{aligned}\ddot{x}_1&=\phi_1 H\sin\omega t+\psi_1 P_1-\frac{R_{x1}}{m_k},\\ \ddot{z}_1&=\phi_2 H\sin\omega t+\psi_2 P_1-\frac{R_{z1}}{m_k}-g.\end{aligned}\right\} \tag{12}$$

Let us integrate the set of differential equations (12). After twofold integration and finding the arbitrary constants we obtain the following solutions of the differential equations (4) in the final form:

$$\left.\begin{aligned}\dot{x}_1&=-\frac{\phi_1 H}{\omega}\cos\omega t+\psi_1 P_1 t-\frac{R_{x1}t}{m_k}+\frac{\phi_1 H}{\omega},\\ \dot{z}_1&=-\frac{\phi_2 H}{\omega}\cos\omega t+\psi_2 P_1 t-\frac{R_{z1}t}{m_k}-gt+\frac{\phi_2 H}{\omega}.\end{aligned}\right\} \tag{13}$$

$$\left.\begin{aligned}x_1&=-\frac{\phi_1 H}{\omega^2}\sin\omega t+\frac{\psi_1 P_1 t^2}{2}-\frac{R_{x1}t^2}{2m_k}+\frac{\phi_1 Ht}{\omega}+x_{10},\\ z_1&=-\frac{\phi_2 H}{\omega^2}\sin\omega t+\frac{\psi_2 P_1 t^2}{2}-\frac{R_{z1}t^2}{2m_k}-\frac{gt^2}{2}+\frac{\phi_2 Ht}{\omega}-\frac{1}{3}h_k.\end{aligned}\right\} \tag{14}$$

The sets of equations (13) and (14) describe the laws of the change in speed and the shift of the centre of the root crop mass during its direct extraction from soil. From the second equation of the set (14) it is possible to define the time t of the root crop direct extraction from soil. For this purpose it is necessary to substitute in the left part of the specified equation value z_1=0 and to solve the obtained equation in relation to t. As the equation is transcendental, it is impossible to obtain an analytical expression for the definition t; nevertheless it can be solved on the computer by means of the known numerical methods. The calculated mean t_1 can be applied to the definition of the unit productivity for the root crop extraction by means of vibratory digging tools.

Let us solve the set of differential equations (5). To simplify the recording of the given set, let us write:

$$\frac{1}{m_k}\left(\frac{2\sin\gamma\, tg\gamma}{\sqrt{tg^2\gamma+1+tg^2\beta}}+2f\sin^3\gamma\cos\delta+f\sin 2\gamma\cos\gamma\right)=\psi_1', \tag{15}$$

$$\frac{1}{m_k}\left(\frac{2\sin\gamma\, tg\beta}{\sqrt{tg^2\gamma+1+tg^2\beta}}-2f\sin^2\gamma\sin\delta\right)=\psi_2'. \tag{16}$$

In view of the expressions (15), (16) the set of differential equations (5) will assume the form:

$$\left.\begin{aligned} \ddot{x}_1 &= \psi_1' P_1 - \frac{R_{x_1}}{m_k}, \\ \ddot{z}_1 &= \psi_2' P_1 - \frac{G_k}{m_k} - \frac{R_{z_1}}{m_k}, \end{aligned}\right\} \tag{17}$$
$$\omega t \in [(2k-1)\pi,\ 2k\pi,\ ],\ k = 1, 2, ...$$

After twofold integration of the set of equations (17) and finding the arbitrary constants we will obtain the final set of differential equations (5) in the final form:

$$\left.\begin{aligned} \dot{x}_1 &= \psi_1' P_1 t - \frac{R_{x_1}}{m_k} t, \\ \dot{z}_1 &= \psi_2' P_1 t - \frac{G_k}{m_k} t - \frac{R_{z_1}}{m_k} t, \end{aligned}\right\} \tag{18}$$
$$\omega t \in [(2k-1)\pi,\ 2k\pi,\ ],\ k = 1, 2, ...$$

$$\left.\begin{aligned} x_1 &= \psi_1' P_1 \frac{t^2}{2} - \frac{R_{x_1} t^2}{2m_k} + x_{10}, \\ z_1 &= \psi_2' P_1 \frac{t^2}{2} - \frac{G_k t^2}{2m_k} - \frac{R_{z_1} t^2}{2m_k} - \frac{1}{3} h_k, \end{aligned}\right\} \tag{19}$$
$$\omega t \in [(2k-1)\pi,\ 2k\pi,\ ],\ k = 1, 2, ...$$

The sets of equations (18) and (19), accordingly, describe the laws of the change in speed and the movement of the centre of the root crop mass during their direct extraction from soil in the absence of the impact force.

Let us set up a differential equation of the root crop rotation around their center of mass, or around the conditional axis Cy_c which passes through the centre of the mass (point C) on the parallel axis O_1y_1. According to the equation it will have a general form:

$$I_{y_c} \frac{d^2\theta}{dt^2} = M_{y_c}^{e}, \tag{20}$$

where θ – angle of the root crop rotation around the axis Cy_c;
I_{yc} – moment of inertia of the root crop in relation to the axis Cy_c;
$M_{y_c}^{e}$ – moment of rotation around the axis Cy_c (the sum of the moments of all external forces which act upon the root crop, in relation to the axis Cy_c).

The moment of inertia I_{yc} of the root crop in relation to the axis Cy_c is defined according to such expression:

$$I_{y_c} = \left(\frac{3}{80} + \frac{3}{20} tg^2 \varepsilon \right) m_k h_k^2 . \tag{21}$$

After substituting the expressions (2), (21) in the differential equation (20) and carrying out the necessary transformations we shall obtain the differential equation of the rotation of the root crop around the axis Cy_c during direct vibrational extraction from the soil (i.e. at the action of the impact force on it) which has the form:

$$\left(\frac{3}{80}+\frac{3}{20}tg^{2}\varepsilon\right)m_k h_k^2\frac{d^2\theta}{dt^2}=-H\left(-h_k+h-z_1\right)\sin\theta\sin\omega t+2P_1\cos\theta\left(-h_k+h-z_1\right)+$$
$$+2\left(\frac{1}{2}fH\cos\delta\sin\omega t+fP_1\sin\gamma\right)\sin\left(\gamma+\alpha_{K1\ \max}\sin\omega t\right)\cos\varepsilon\left(-h_k+h-z_1\right)\sin\theta+ \qquad (22)$$
$$+2\left(\frac{1}{2}fH\cos\delta\sin\omega t+fP_1\sin\gamma\right)\cos\left(\gamma+\alpha_{K1\ \max}\sin\omega t\right)\cos\gamma\left(-h_k+h-z_1\right)\cos\theta-M,$$
$$\omega t\in\left[2k\pi,\ (2k+1)\pi\right],\quad k=0,1,2,...$$

The differential equation of the root crop rotation around the axis Cy_c at usual extraction (i.e. in the absence of impact force), has the form:

$$\left(\frac{3}{80}+\frac{3}{20}tg^{2}\varepsilon\right)m_k h_k^2\frac{d^2\theta}{dt^2}=2P_1\cos\theta\left(-h_k+h-z_1\right)+2fP_1\sin^2\gamma\times$$
$$\times\cos\varepsilon\left(-h_k+h-z_1\right)\sin\theta+fP_1\sin2\gamma\cos\gamma\left(-h_k+h-z_1\right)\cos\theta-M, \qquad (23)$$
$$\omega t\in\left[(2k-1)\pi,\quad 2k\pi\right],\qquad k=1,2,...$$

Let us analyze the obtained differential equations (22) and (23). The differential equation (22) is nonlinear. It is possible to solve it by approximated numerical methods using a computer, and in each step of the application of a numerical algorithm it is necessary to find the value z_1 from the second equation of the set (14) for the corresponding moment of time t_k. The differential equation (23), which includes a variable quantity z_1, is also nonlinear, and for each moment of time t_k it is necessary to define the value z_1 from the second equation of the set (19).

Thus, it is finally possible to regard that a mathematical model is developed of the process of direct sugar beet extraction from soil by vibrational digging. The obtained results enable to define kinematic modes of the root crop vibrational digging considering the constructive parameters of the vibratory digging tools.

Conclusions

1. Two sets of differential equations describe plane-parallel motion of a root crop in soil during its direct extraction carried out by interaction of the vertical impact force, which is transmitted the root crop from the vibratory digging tool, and the tractive effort, which arises owing to longitudinal movement of the digger.
2. These differential equations provide an opportunity to find out the law of the root crop movement in a longitudinal-vertical plane during their direct extraction from soil.
3. The obtained results enable also to define the kinematic modes of the root crop vibrational digging without causing damage to the roots and to find rational constructive parameters of the vibrational digging tool.

References

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