MATHEMATICAL SIMULATION OF OSCILLATIONS OF TOWED AGRICULTURAL AGGREGATES

Volodimir Bulgakov[1], Semjons Ivanovs[2]
[1]National Agrarian University of Ukraine,
[2]Latvia University of Agriculture, Research Institute of Agricultural Machinery
semjons@delfi.lv

Abstract. Oscillatory movements of towed agricultural machine aggregates have been studied and analysed in the process of their movement over the surface irregularities of soil. Differential equations have been worked out of the movement of mechanical systems in a longitudinal vertical plane with two and with one degree of freedom.

Keywords: mechanical system, agricultural aggregates, oscillations.

Introduction

The quality of the technological process performed by agricultural machines and machine aggregates depends to a certain degree on the stability of their movement in a vertical plane when they are running over the surface irregularities of soil. This particularly concerns the towed machine aggregates that are complex dynamic systems, i.e., aggregates consisting of a tractor, a towed combine harvester and, in some cases, a trailer for collecting the harvested product. A mobile agricultural machine aggregate can be a mobile undercarriage (or a tractor) onto which all the operating tools of the combine harvester are hanged. Since the operating tools of a mobile machine aggregate are hanged onto the undercarriage (tractor), then it can be regarded as a joint mobile oscillatory system.

The previous investigations on the present topic [1, 2] contain no detailed analytical study of the towed machine aggregates composed in different patterns.

Materials and methods

The aim of this investigation is to establish the degree of the impact of oscillatory movements of agricultural machine aggregates on the quality indices of their work. In order to reach the advanced aim, theoretical studies were applied based on differential equations of the movement of a mechanical system using the Lagrange equations of the 2^{nd} kind.

Results and discussion

Let us build an equivalent calculation scheme of the oscillatory movements of a towed agricultural machine aggregate (Fig. 1.). It will be a mechanical system with one degree of freedom.

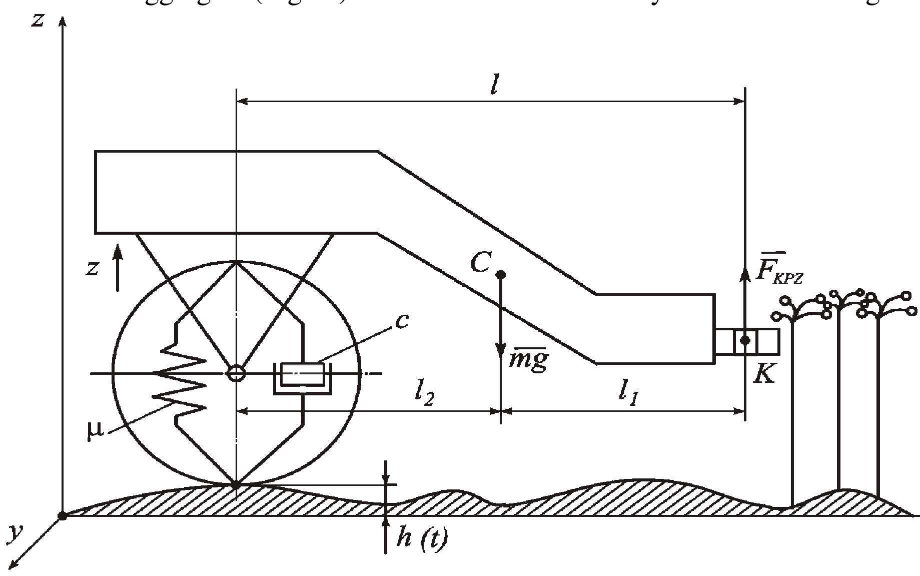

Fig. 1. **An equivalent scheme of a machine aggregate reduced to an oscillatory mechanical system with one degree of freedom**

Letter designations: *Oxyz* – a fixed system of coordinates (plane *xOz* is a vertical plane which is perpendicular to the plane of the field), *z* – vertical displacement of the centre of mass of the aggregate; *C* – centre of masses of the system; l_1 – the distance from the frontal point of the suspension to the centre of masses of the system; l_2 – the distance of the rear point of the suspension to the centre of masses of the system; *l* – the distance from the frontal point to the rear point of the suspension of the system, *c* – suspension stiffness of the machine aggregate; *h* – the height of the irregularity of the bearing surface of the soil under the wheels, μ – the resistance coefficient of the suspension and the tyres of the machine aggregate.

We take the vertical movement *z* of a sprung mass over the rear running wheels (there are no front wheels) as a generalised coordinate. We will start counting off the generalised coordinate *z* from the position of static equilibrium of the system. Then the movement of this mechanical system will also be described in the form of the Lagrange equation of the 2nd kind:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{z}}\right) - \frac{\partial T}{\partial z} = Q_z, \quad (1)$$

where $T = \frac{1}{2} m \dot{z}^2$;

$\Pi = \frac{1}{2} c (z - h)^2$;

$h = h(t)$;

$\Phi = \frac{1}{2} \mu (\dot{z} - \dot{h})^2$;

$m = \frac{M\, l_1}{l}$ – the mass of a part of the machine aggregate which performs the vertical oscillatory movements.

Let us determine the generalised force for this instance of the movement of the machine aggregate. It will be equal to:

$$Q_z = Q_z^{(\Pi)} + Q_z^{(\Phi)} + Q_z^{(B)}, \quad (2)$$

where

$$Q_z^{(\Pi)} = -\frac{\partial \Pi}{\partial z} = -c(z - h),$$

$$Q_z^{(\Phi)} = -\frac{\partial \Phi}{\partial \dot{z}} = -\mu(\dot{z} - \dot{h}),$$

$$Q_z^{(B)} = 0,$$

$$Q_z = -c(z - h) - \mu(\dot{z} - \dot{h}).$$

We perform the necessary transformations for (1).

We have:

$$\frac{\partial T}{\partial \dot{z}} = m\dot{z},$$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{z}}\right) = m\ddot{z},$$

$$\frac{\partial T}{\partial z} = 0.$$

By substituting (2) into (1) and using the performed transformations we obtain:

$$m\ddot{z} = -c(z-h) - \mu(\dot{z} - \dot{h}),$$

or:

$$\ddot{z} = -\frac{c}{m}(z-h) - \frac{\mu}{m}(\dot{z} - \dot{h}),$$

or:

$$\ddot{z} + \frac{\mu}{m}\dot{z} + \frac{c}{m}z = \frac{ch}{m} + \frac{\mu}{m}\dot{h}, \tag{3}$$

where

$$\frac{c}{m} = k^2,$$

$$\frac{\mu}{2m} = n \cdot$$

Then the differential equation (3) will assume the following appearance:

$$\ddot{z} + 2n\dot{z} + k^2 z = k^2 h(t) + 2n\dot{h}(t). \tag{4}$$

It is known that the common solution of the differential equation (4) is equal to:

$$z = z_1 + z_2, \tag{5}$$

where z_1 – the common solution of the homogeneous differential equation

$$\ddot{z} + 2n\dot{z} + k^2 z = 0, \tag{6}$$

z_2 – a partial solution of the nonhomogeneous differential equation depending on the kind of the right side.

According to the theory of differential equations, the common solution of the differential equation (6) can have one of the following appearances:

If there is low resistance ($n<k$):

$$z_1(t) = e^{-nt}(C_1 \cos k_1 t + C_2 \sin k_1 t),\ k_1 = \sqrt{k^2 - n^2}, \tag{7}$$

or

$$z_1(t) = ae^{-nt} \sin(k_1 t + \beta).$$

If there is high resistance ($n>k$):

$$z_1(t) = e^{-nt}\left(C_1 e^{k_2 t} + C_2 e^{-k_2 t}\right),\ k_2 = \sqrt{n^2 - k^2}. \tag{8}$$

If there is critical resistance ($n=k$):

$$z_1(t) = e^{-nt}(C_1 + C_2 t). \tag{9}$$

In expressions (7-9) C_1 and C_2 – arbitrary constants, which are determined from the initial conditions.

Instances 2 and 3 are fading nonoscillatory movements. Instance 1 – free fading oscillations with amplitude ae^{-nt} frequency k_1.

Structure $z_2(t)$ of a partial solution of the differential equation (1) depends on the surface irregularities of soil, i.e., on the kind of function $h(t)$.

The irregularities of soil can be described with a certain approximation by the following harmonic function:

$$h(t) = h_o \sin\left(\frac{Vt}{L}\right), \tag{10}$$

where $h(t)$ – the height of the irregularities of soil;
L – the length of the irregularities of soil;
V – the forward speed of the movement of the machine aggregate;
($h_\circ$, V, L) – the parameters the values of which are assigned.

Let us define further

$$\frac{V}{L} = k_3 .$$

Then expression (10) will assume such an appearance:

$$h(t) = h_o \sin k_3 t . \tag{11}$$

By substituting expression (11) into the differential equation (4) we obtain:

$$\ddot{z} + 2n\dot{z} + k^2 z = k^2 h_o \sin k_3 t + 2nh_o k_3 \cos k_3 t . \tag{12}$$

Then the partial solution z_2 of equation (12) should be searched for in the following way:

$$z_2 = A\sin k_3 t + B\cos k_3 t , \tag{13}$$

where A, B – unknown coefficients.

Let us determine coefficients A and B by the method of uncertain coefficients. For this we find the necessary derivatives:

$$\dot{z}_2 = Ak_3 \cos k_3 t - Bk_3 \sin k_3 t , \tag{14}$$

$$\ddot{z}_2 = -Ak_3^2 \sin k_3 t - Bk_3^2 \cos k_3 t . \tag{15}$$

By substituting expressions (14) and (15) into (12) we obtain:

$$\begin{aligned} &- Ak_3^2 \sin k_3 t - Bk_3^2 \cos k_3 t + 2nAk_3 \cos k_3 t - \\ &- 2nBk_3 \sin k_3 t + k^2 A \sin k_3 t + \\ &+ k^2 B \cos k_3 t = k^2 h_o \sin k_3 t + 2nh_o k_3 \cos k_3 t. \end{aligned} \tag{16}$$

By equating the coefficients at equal functions in the left and the right side of expression (16) we obtain the following system of algebraic equations in relation to the unknown A and B:

$$\left.\begin{aligned} - Ak_3^2 - 2nBk_3 + k^2 A = k^2 h_o , \\ - Bk_3^2 + 2nAk_3 + k^2 B = 2nh_o k_3 . \end{aligned}\right\} \tag{17}$$

We apply the Cramer method to solve the system of equations (17), and therefore we rewrite this system in the following way:

$$\left.\begin{aligned} \left(k^2 - k_3^2\right)A - 2nk_3 B = k^2 h_o , \\ 2nk_3 A + \left(k^2 - k_3^2\right)B = 2nh_o k_3 . \end{aligned}\right\} \tag{18}$$

Let us calculate the necessary determiners:

$$\Delta = \begin{vmatrix} k^2 - k_3^2 & -2nk_3 \\ 2nk_3 & k^2 - k_3^2 \end{vmatrix} = \left(k^2 - k_3^2\right)^2 + 4n^2 k_3^2,$$

$$\Delta_A = \begin{vmatrix} k^2 h_o & -2nk_3 \\ 2nh_o k_3 & k^2 - k_3^2 \end{vmatrix} = h_o \left[k^2 \left(k^2 - k_3^2\right) + 4n^2 k_3^2 \right],$$

$$\Delta_B = \begin{vmatrix} k^2 - k_3^2 & k^2 h_o \\ 2nk_3 & 2nh_o k_3 \end{vmatrix} = -2nk_3^3 h_o .$$

Then:

$$A = \frac{\Delta_A}{\Delta} = \frac{h_o \left[k^2 \left(k^2 - k_3^2\right) + 4n^2 k_3^2 \right]}{\left(k^2 - k_3^2\right)^2 + 4n^2 k_3^2}, \tag{19}$$

$$B = \frac{\Delta_B}{\Delta} = -\frac{2nk_3^3 h_o}{\left(k^2 - k_3^2\right) + 4nk_3^2} . \tag{20}$$

Consequently, a partial solution $z_2(t)$ follows from expression (13) where coefficients A and B are deduced from expressions (19) and (20), respectively. It is known that expression (13) can be written in the following way:

$$z_2(t) = H \sin\left(k_3 t + \beta_3\right), \tag{21}$$

where

$$H = \sqrt{A^2 + B^2}\,,\ \mathrm{tg}\beta_3 = \frac{B}{A} . \tag{22}$$

Expression (21) describes the forced oscillations of the agricultural machine aggregate in a longitudinal vertical plane with amplitude H and frequency k_3.

Besides, the number

$$\beta_3 = \mathrm{arctg}\frac{B}{A} \tag{23}$$

is the initial stage of the forced oscillations of the machine aggregate.

Thus, taking into account (5), the common solution of the differential equation (12) can be written in one of the following forms:

If there is low resistance ($n<k$):

$$z(t) = e^{-nt}\left[C_1 \cos k_1 t + C_2 \sin k_1 t\right] + A \sin k_3 t + B \cos k_3 t ,$$

or

$$z(t) = a e^{-nt} \sin\left(k_1 t + \beta\right) + H \sin\left(k_3 t + \beta_3\right). \tag{24}$$

If there is high resistance ($n>k$):

$$z(t) = e^{-nt}\left(C_1 e^{k_2 t} + C_2 e^{-k_2 t}\right) + A \sin k_3 t + B \cos k_3 t ,$$

or

$$z(t) = e^{-nt}\left(C_1 e^{k_2 t} + C_2 e^{-k_2 t}\right) + H \sin\left(k_3 t + \beta_3\right). \tag{25}$$

If there is critical resistance $n=k$:

$$z(t) = e^{-nt}(C_1 + C_2 t) + A\sin k_3 t + B\cos k_3 t\,,$$

or

$$z(t) = e^{-nt}(C_1 + C_2 t) + H\sin(k_3 t + \beta_3). \tag{26}$$

The arbitrary constants C_1 and C_2 are determined from the following initial conditions at $t = 0$:

$$z = 0,\ \dot{z} = 0. \tag{27}$$

If there is high or critical resistance, then the nonoscillatory movements fade rather quickly, and therefore at $t>T$, where T – a certain moment of time, one can consider that

$$z(t) \approx H\sin(k_3 t + \beta_3), \tag{28}$$

i.e., the oscillations of the machine aggregate take place only at the expense of forced oscillations. In the case of low resistance ($n<k$) the free fading oscillations can essentially influence the oscillatory process of the towed machine aggregate.

Since in the case of low resistance ($n<k$) the entire oscillatory process of the towed machine aggregate is described by the differential equation (24), we find out the arbitrary constants C_1 and C_2 from the initial conditions (27).

By differentiation of expression (24) in time t we obtain:

$$\begin{aligned} \dot{z}(t) &= -ne^{-nt}(C_1\cos k_1 t + C_2\sin k_1 t) + \\ &+ e^{-nt}(-k_1 C_1\sin k_1 t + k_1 C_2\cos k_1 t) + \\ &+ k_3 A\cos k_3 t - k_3 B\sin k_3 t. \end{aligned} \tag{29}$$

Taking into account the initial conditions (28), we will obtain a system of algebraic equations in relation to the unknown C_1 and C_2:

$$\left.\begin{aligned} -nC_1 + k_1 C_2 + k_3 A = 0, \\ C_1 + B = 0. \end{aligned}\right\} \tag{30}$$

From the obtained system of equations we find:

$$C_1 = -B,\ C_2 = -\frac{nB + k_3 A}{k_1}. \tag{31}$$

By substituting the values of C_1 and C_2 from (31) into expression (24) we obtain a rule of the oscillatory movements of the towed machine aggregate in a vertical plane arising from the impact of soil irregularities:

$$\begin{aligned} z(t) &= -e^{-nt}\left(B\cos k_1 t + \frac{nB + k_3 A}{k_1}\sin k_1 t\right) + \\ &+ A\sin k_3 t + B\cos k_3 t\,, \end{aligned} \tag{32}$$

where A, B – coefficients are deduced from expressions (19) and (20), respectively.

Let us write down expression (32) in the following way:

$$z(t) = -\alpha e^{-nt}\sin(k_1 t + \beta) + H\sin(k_3 t + \beta_3), \tag{33}$$

where

$$\alpha = \sqrt{B^2 + \frac{(nB + k_3 A)^2}{k_1^2}},\ \beta = arctg\,\frac{k_1 B}{nB + k_3 A}, \tag{34}$$

H and β_3 – determined according to expressions (22) and (23), respectively.

Conclusions

Application of the theory of oscillatory movements provides a possibility to find in an analytical way the stabilisation conditions of the movement of towed agricultural machine aggregates in a longitudinal vertical plane, which, in their turn, will lead to higher quality of the performed operations.

References

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